THE EFFECT OF POLYMER ADDITIVES ON OSCILLATORY FLUID FLOW

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ABSTRACT

The effects of drag-reducing polymer additives on oscillatory fluid flow, particularly the damping of small-amplitude gas bubble pulsation and liquid column oscillation, were investigated.

The damping of bubble pulsation was investigated both analytically and experimentally. The analytical investigation showed that the elasticity of the visco-elastic liquids opposed the viscous effects on the damping of bubble pulsation. On the experimental side, an apparatus was designed to produce gas bubbles of uniform size. The performance of the apparatus was satisfactory. The experimental results of the effects of Polyox additives on the damping of bubble pulsation were in agreement with the theoretical analysis. However, the results of the effects of the Separan additives would support the presence of anomalous extensional viscosity effects on the damping of bubble pulsation. The effects of the polymer additives on bubble formation and sound radiation was investigated.

The effects of the polymer additives on the damping of liquid column oscillation were investigated experimentally. It was found that the steady-state rheological properties of the visco-elastic liquids can adequately describe their liquid column oscillating flow. The absence of the effects of the solutions elasticity is attributed to the smallness of their relaxation time in comparison with the time period of the liquid column oscillation.

CHAPTER 1

INTRODUCTION

1.1 GENERAL STATEMENT

Drag reduction is the reduction of skin friction in turbulent flow below that of the solvent alone. The reduction of turbulent drag, by long chain, high molecular weight polymer additives in turbulent shear flow has been the subject of intensive research in the past two decades. Other phenomena associated with polymer additives in non turbulent shear flow fields have also received some attention. Various mechanisms have been proposed to explain the phenomenon of drag reduction. So far, none have proved to be fully successful. The difficulties that face researchers in drag reduction stem from the fact that neither turbulence nor rheology of polymer solutions are yet fully understood. The potential engineering applications of drag-reduction are very large but they would only be realised when the phenomenon is fully understood.

In this work, the effects of drag-reducing polymer additives on oscillatory fluid flow are investigated. Two fluid flows are looked at: the free pulsation of air bubbles in liquids and the free oscillation of liquid columns.

1.2 SURVEY OF LITERATURE

In the following sections of this chapter, we will present some of the views expressed in the literature concerning the topics relevant to this work. These will be presented in the following order:

1. Turbulent drag-reduction by polymer additives.

2. Hypotheses of drag-reduction by polymer additives.

3. The effects of polymer additives on non turbulent shear flow.

4. Cavitation and bubble dynamics in Newtonian and visco-elastic liquids.

5. Free oscillation of Newtonian and visco-elastic liquid column.

1.3 TURBULENT DRAG-REDUCTION BY POLYMER ADDITIVES

The effects of drag-reducing polymer additives on internal and external turbulent shear flow have been investigated by very many researchers. Several reviews of the subject have been published and cover several aspects of the drag-reduction phenomenon (Lumley (1969), Hoyt (1972), Virk (1975)). In this section we present the effects of polymer additives on some aspects of turbulent shear flow in pipes. However, before doing that, the effects of the molecular properties of the polymer additives on the drag-reduction are worth mentioning.

1.3.1 The Effect of the Molecular Properties of Polymer Additives on Drag-Reduction

Different polymers are in use as drag-reducers (Polyethelyne Oxide, Polyacrylamides, Guargum). These polymers have different molecular properties (molecular weight, molecular structure, solubility, etc). The molecular properties of the polymers affect the drag-reduction quantitatively, as least, and possibly qualitatively.

Polymers of high molecular weights have been observed to be more effective as drag-reducers than low molecular weight polymers. To attain the same percentage of drag-reduction under the same conditions, a lower concentration of the high molecular

weight polymer is required than of the low molecular weight polymer (Arunachalam and Fulford (1971)). Polymers having a linear structure with few or no side chains were observed to be more efficient than polymers with many branched side chains (Lumley (1969)). Polymers when dissolved in good solvents (high polymer-solvent interaction) were found to be more effective in reducing turbulent drag than when dissolved in poor solvents (Hoyt (1972)). Polymers of flexible structure were observed to be more effective in reducing turbulent drag than polymers of rigid structure (Lumley (1969)).

All polymers show a concentration-dependent increase in turbulent drag-reduction until a maximum is reached. Thereafter upon increasing the concentration, a decline in the drag-reduction percentage is observed. The concentrations at which maximum drag-reduction occurs differ for different polymers and molecular weights (Hoyt (1972)).

1.3.2 Onset of Drag Reduction

It is widely accepted by researchers in drag-reduction that this phenomenon onsets at a critical value of wall shear stress (Virk et al (1967), Little et al (1975)). Below this critical value, no drag-reduction was observed. The explanation of the onset of drag-reduction is still a subject of controversy. Two hypotheses have been proposed: A time-scale hypothesis (Lumley (1973)) and a length scale hypothesis (Virk (1975)). The experimental results are so diverse that both parties claim validity of their hypotheses. (For details see Lumley (1973) and Virk (1975).)

The time scale hypothesis argues that turbulent drag onsets when the time scale of the turbulent eddies become comparable with the relaxation time of the polymer molecule. The length scale hypothesis argues that drag reduction onsets when the radius of gyration of the polymer macro-molecule attains a certain size.

Experimental evidence showed that several parameters affect the onset phenomena. Little et al (1975) observed that the value of critical shear stress decreased with an increase in the polymer concentration of Polyox WSR-205. Whitsitt et al (1969) showed a concentration-dependence of the critical shear stress for polyacrylamides. Goren and Norbury (1967) and Virk (1971) showed no concentration effect for Polyox WSR-301. The molecular weight of the polymer seems relevant to the onset phenomenon. Little et al (1975) reported that the critical shear stress required to onset drag-reduction increased with the molecular weight. Hoyt (1973) is of the opinion that substances with highest molecular weight have lowest threshold values. Pipe diameter seems to have little effect on the onset phenomenon (Little et al (1975), Virk (1975).

However, Hoyt (1972) proposed a reconciliation of the time scale hypothesis and length scale hypothesis by postulating that the polymer acts as aggregates of molecules to give a physical size comparable with small turbulent eddies but with relaxation time a function of the individual molecule.

1.3.3 Velocity Profile

Velocity profiles in pipe flow of dilute polymer solutions

were measured by pitot tubes, hot-wire probes and laser-doppler anemometers. The pitot tubes and hot-wire probes were unsuccessful due to the errors involved in the measurements. These errors are attributed to the effects of polymer solutions on the functioning of these instruments. Hot-wires become less sensitive than in Newtonian liquids. Pitot tubes give low readings (Astarita and Nicodemo (1969)).

Laser-doppler anemometers proved the most successfull in measuring the velocity profile and turbulence structures in polymer solutions. Velocity profiles obtained by using laser-doppler techniques demonstrated a thickening of the viscous sub-layer next to the wall (Rudd (1969)). According to Virk (1975) the thickening of the viscous sub-layer is characterised by the appearance of a region lying between the viscous sub-layer and the outer Newtonian plug. Virk called this layer the elastic sub-layer.

In the viscous sub-layer, no changes in the velocity profile have been observed. In the turbulent plug, the velocity profile is observed to be parallel-shifted upwards in comparison with the Newtonian plug. In the elastic sub-layer, an effective velocity slip is observed leading to the higher values of the velocity in the turbulent plug (Virk (1975), Rollin and Seyer (1972), Patterson (1969)).

1.3.4 Turbulence Structure

Reliable data on the structure of turbulence in drag-reducing polymer solutions were obtained by using laser-doppler anemometers. Other techniques, pitot tubes, hot-wire probes and small air bubbles seemed to have failed to give correct data. Seyer and

Metzner (1969) used small air bubbles to study turbulence structure in drag-reducing solutions. They found that the axial component of turbulence was reduced by the polymer additives. No changes in the radial component of turbulence were observed.

Later studies of turbulence structure by using laser-doppler anemometers (Rudd (1969), Logan (1972)) showed that the axial component of turbulence varied in a complicated manner, the axial component was slightly reduced in the centre of the tube, whereas in the wall region it increased above the Newtonian value. Radial turbulence intensity in the core of the flow was observed to be the same in both Newtonian and polymer solutions. In the wall region, a slight reduction (10%) was observed (Virk (1975)).

1.4 HYPOTHESIS PROPOSED TO EXPLAIN DRAG-REDUCTION

Several hypotheses have been offered by different researchers to explain the phenomenon of drag-reduction by polymer additives (Arunachalam and Fulford (1971), Hoyt (1972)). So far, none of these hypotheses have proved to offer a full qualitative and quantitative explanation of the phenomenon. In this section, we present some of these hypotheses.

1. Shear-thinning Hypothesis

Toms (1949), after whom the drag-reduction phenomenon is sometimes named, suggested that the addition of polymer in turbulent shear flow will result in the creation of a shear-thinning wall layer which, by virtue of low viscosity, would give a reduction in the friction factor of the polymer solution compared with the pure solvent. Measurements of the viscosity of

some drag-reducing polymers (Polyox and Guargum) at concentrations giving optimum drag-reduction showed that these polymers are not shear thinning (Hoyt (1972). Walsh (1967) obtained drag reduction in turbulent shear flow by using shear thickening substances (see Hoyt (1972)). Consequently, this hypothesis does not seem to be valid.

2. Wall Adsorption Hypothesis

This hypothesis postulates that polymer molecules would be adsorbed on the wall adjacent to the flow. The adsorbed layer of polymer would then interact with the flow and ultimately reduce turbulent drag. It was postulated (Arunachalam and Fulford (1971)) that loops of polymer molecules adsorbed on the wall would protrude in the flow and modify the usual boundary layer. The interaction of the polymer molecules with the boundary layer will reduce the velocity gradient in the layer adjacent to the wall. Turbulent drag is therefore reduced. The available experimental results are conflicting; some support this hypothesis and some disprove it. The experimental results of El'perin and Smolskii (1965), Davis and Ponter (1976), Hand and Williams (1973) and Ayyash and McComb (1976) offer some support for this hypothesis. The measurements made by Gyr and Mueller (1974) and Little et al (1975) suggest that the adsorbed layers of polymers have very little effect - if any - in the drag-reduction phenomenon.

3. Non-Isotropic Viscosity Hypothesis

This hypothesis postulates that the interaction of the polymer molecules with the solvent in turbulent shear flow will

result in non-isotropic viscosity of the polymer solution. The viscosity would be low in the direction of the flow and high enough in the other directions to dampen the oscillations that contribute to the generation and growth of turbulence (Hoyt (1972)). Gadd's measurements (1966) of the normal stresses in Polyox, Separan and Guargum solutions showed that Polyox solutions only exhibited substantial differences in the stress components. Other drag-reducing solutions did not exhibit such differences. Therefore, a universal theory cannot be based on this hypothesis. McComb (1973) proposed this hypothesis to explain the drag-reduction by fibre suspensions in turbulent shear flow.

4. Visco-elasticity Hypothesis

Several hypotheses have been postulated attributing the phenomenon of drag-reduction to the effects of the visco-elastic properties of the polymers on the structure of turbulence in shear flow. Astarita (1965) suggested that turbulence in polymer solutions is less dissiptative than in purely viscous solutions. Gadd (1965, 1966) suggested that drag-reduction is associated with reduced generation of turbulence. Gadd's hypothesis is based on the fact that polymer solutions exhibit high resistance to stretching, therefore, the stretching of vortices would be reduced. Consequently, reduced mixing and faster damping of the turbulent eddies occur. This leads to a reduced generation of turbulence and a reduction in turbulent drag. This hypothesis is supported by the high resistance to stretching shown by dilute solutions of drag-reducing polymers (Metzner and Metzner (1970), Balakrishnan and Gordon (1975)).

However, the hypothesis most favoured among researchers postulates that the polymer molecules interact with the "bursting" process responsible for the generation of turbulence. Kim et al (1971) revealed that "bursting" is responsible for the creation of most of the turbulent stresses. Donohue et al (1972) found that polymer additives decreased the number of bursts per unit area and also reduced the intensity of the burst. Virk (1975) suggested that the polymer molecules interfere with the turbulent bursting process and that at the onset of drag-reduction, the duration of a turbulent burst is of the order of the relaxation time of a macro-molecule. Little et al (1975) proposed that the bursting phenomenon is associated with an oscillatory disturbance super-imposed on the steady shear flow motion. It is postulated that the polymer additives will increase the decay rate of such oscillatory disturbances and thereby decrease bursting and the associated generation of turbulence.

1.5 THE EFFECT OF POLYMER ADDITIVES ON NON-TURBULENT FLUID FLOW

The universally accepted fact among researchers in drag-reduction is that polymer additives reduce turbulent drag in turbulent shear flow in comparison with pure Newtonian solvents. In non-turbulent shear flow, polymer solutions can behave in a non-Newtonian way as well. However, instead of reducing the flow drag, they increase it in many cases. The increase in drag is more than to be accounted for by the increase in viscosity due to the additives.

In this section we look at the behaviour of drag-reducing polymer solutions in some non-turbulent flows. These flows have industrial applications which are not of concern to us here. The flow of polymer

solutions in non-turbulent shear flow presents beneficial information to the researchers in drag-reduction which may help in explaining the drag-reduction phenomenon. Below, we look at three different types of flow:

1. Orifice flow of Polymer solutions.
2. Flow of polymer solutions in porous media.
3. Extensional flow of polymer solutions.

1.5.1 Orifice Flow of Polymer Solutions

An interesting aspect of the flow of drag-reducing polymer solutions in small orifices is the non-Newtonian behaviour exhibited by these solutions at low concentrations which does not affect the viscometric flow of these solutions. This non-Newtonian behaviour is exhibited in terms of increasing pressure drop that cannot be attributed to simple increase in viscosity. The submerged laminar jets of dilute polymer solutions ~~we~~ are reported to expand more than those of Newtonian liquids (Bilgen (1973)). Except for a report by Bate (1967), most of the reported observations on the flow of polymer solutions - to the knowledge of the author - showed an increase in the pressure drop across a small nozzle. Bate (1967) reported that Polyox WSR-301 additives reduced the pressure drop across an orifice plate.

Giles (1969) reported increased pressure drop in orifice flow of Polyox coagulant solutions. The same effect was reported for Polyox WSR-301 solutions. However, solutions of Guargum at concentrations giving maximum drag-reduction in turbulent shear flow behaved in a Newtonian way in orifice flow. Morgan (1971) conducted a series of experiments using different polymers, ionic and non-ionic, in pure and salted water. He observed an

increase in the pressure drop in the laminar flow regime of these solutions. The addition of NaCl to ionic polymer reduced the amount of pressure drop. The abnormal behaviour of the polymer solutions is attributed to their elasticity. However, the failure of Guargum solutions to exhibit such elastic effects and the reduction in the elasticity of the ionic polymers due to the addition of NaCl cast some doubt on the importance of elasticity in drag-reduction (Hoyt (1972)). Bilgen (1973) studied the behabiour of Polyox solutions of varying molecular weights and concentrations in the laminar regime. Bilgen found that the nozzle discharge coefficients of dilute polymer solutions (except WSR-205) were considerably smaller than those for water, particularly at low Reynolds number. He interpreted the increase in the pressure loss as being due to the elasticity of the solutions.

1.5.2 Flow in Porous Media

The flow of polymer solutions in porous media is important because of its engineering applications in the oil industry. From a drag-reduction point of view, the flow in porous media is interesting because the flowing liquid is subjected to acceleration as well as deceleration. Under such flow configuration elastic effects may appear.

We observed that several researchers have investigated the flow of concentrated polymer solutions in porous media. These solutions can exhibit non-Newtonian behaviour even in simple viscometric flow. Although these investigations are important in the engineering application, they probably are of little benefit to drag-reduction researchers in terms of supplying information

on the properties of dilute solutions of drag-reducing polymers. A recent report by Laufer et at (1976) dealt with the flow of dilute solutions of drag-reducing polymers in porous media.

The flow of polymer solutions in porous media was studied by Christopher and Middleman (1965) using concentrated solutions (> 1%) of Carboxymethycellulose (CMC) and Polyisobutylene (PIB). Both of these polymers are known drag-reducers. Their experimental data correlated well with their model describing the solutions as power-low fluids, ie. viscous non-Newtonian fluids. No visco-elastic effects were noticed. These results were later confirmed by the experimental results of Gaitando and Middleman (1967) using PIB solutions. However, Marshall and Metzner (1967) reported that solutions of Separan AP-273 and PIB exhibited elastic effects in terms of increased pressure drops. The failure of the results of the previous authors to show visco-elastic effects was thought to be due to their flow parameters which were not appropriate to exhibit the visco-elastic effects of the solutions (Marshall and Metzner (1967)).

Laufer et al (1976) studied the behaviour of two dilute polymer solutions, Polyox WSR-301 and Separan AP-273. Both polymers are well-known efficient drag-reducers. The concentrations of the investigated solutions were as low as 25 ppm, a typical concentration for drag-reduction in turbulent shear flow. Laufer et al results showed that the polymer additives increased the pressure drop by 2 - 9 times when compared with Newtonian liquids. The higher the concentration the higher the pressure drop. The authors attributed the increase in the pressure drop to the stretching of the polymer solutions. However, the dilute

solutions of drag-reducing polymer were reported to exhibit high resistance to stretching as presented below.

1.5.3 Extensional Flow of Polymer Solutions

Some of the hypotheses of drag-reduction postulate that the resistance of polymer solutions to stretching will inhibit the stretching of vortices or the growth of bursts. By doing so, lesser generation of turbulence will occur and the turbulent drag will be reduced. Thus, the study of the behaviour of the polymer solutions in extensional flow fields can present valuable information to explain the phenomenon of drag-reduction.

The extensional flow is characterised by the existence of a velocity gradient in the direction of flow. In contrast, in viscometric shear flow no such velocity gradient exists. The viscometric shear flow is characterised by a velocity gradient in the direction normal to the direction of the flow. The extensional viscosity of a Newtonian fluid is three times its shear viscosity (Astarita and Nicodemo (1970)). In polymer solutions, the extensional ~~velocity~~ viscosity can become orders of magnitude higher than the dynamic viscosity. The extensional viscosity of polymer solutions depends strongly on the stretch rate. The extensional viscosity of drag-reducing polymer solutions was measured by Oliver and Bragg (1973, 1974) and Balakrishnan and Gordon (1975) among others. Oliver and Bragg measured the extensional viscosity of solutions of Polyox WSR-301 and Separan AP-273 of concentrations of 700 - 1000 ppm at stretch rates of 100 - 1000 sec^{-1}. Their measurements showed that the extensional viscosity of the solutions can be as high as 10^3 times the shear viscosity. The extensional viscosity increased with the stretch

rate. The results of Oliver and Bragg are interesting because of the stretch rate range they investigated. As will be shown later, the maximum stretch rate of the investigated bubbles in this work lies in the range investigated by Oliver and Bragg.

Balakrishnan and Gordon (1975) investigated the extensional behaviour of a 20 ppm solution of Separan AP-30 at high stretch rates (3800 - 11,000 sec^{-1}). Their results indicated an increase in the extensional viscosity of 1500 - 3000 times the shear viscosity.

Extensional flow of polymer solutions is of interest in studying bubble dynamics in these solutions. The velocity field of a pulsating bubble in liquid is an irrotational one. In polymer solutions, extensional viscosity effects may be exhibited (Ting (1975), Oliver, private communication).

1.6 CAVITATION AND BUBBLE DYNAMICS

The study of bubble and cavitation dynamics is of practical importance. The word 'cavitation' is associated with the mechanical damages induced on the propellers of ships and centrifugal pumps. Gas bubbles in liquids affect the attenuation of sound waves. The vibrations induced by pulsating bubbles can affect the heat and mass transfer in some chemical processes (Houghton (1963)). In drag-reducing polymer solutions, the study of cavitation and bubble dynamics serves two purposes, the effects of the polymer solutions on cavitation dynamics and the associated mechanical damages and the study of the properties of the polymer solutions in non turbulent shear flow.

Cavitation dynamics have received more attention in the past three decades because of their increasing importance in engineering applications and because of the developing of computing facilities

capable of dealing with the non-linear equations of cavitation dynamics and large amplitude bubble oscillation. Several reviews on the dynamics of cavitation and bubbles have been published (Devin (1959), Kapustina (1970), Flynn (1975) and Plesset and Prosperetti (1977)). The reviews covered different aspects of the cavitation and bubble dynamics in Newtonian liquids. A review of the subject in non-Newtonian liquids is still awaited.

The study of cavitation dynamics dates back to more than a century ago. Besant (1859) considered the problem by assuming that a spherical portion of an infinite mass of incompressible fluid is suddenly annihilated. Changes in the dynamics of the fluid were to be found. Lord Rayleigh (1917) presented the first mathematical treatment of the problem by assuming a zero pressure inside the cavity (see Chapter 2). No major advancements in the dynamics of cavitation were achieved until the early fifties.

Gas bubble dynamics were investigated independently of Rayleigh's cavity. Minnaert (1933) investigaged the undamped pulsation of a gas bubble in liquid and derived an expression for the resonance frequency of free pulsating gas bubbles. The thermodynamic field in the gas is the bubble was assumed to be adiabatic. The experimental results of Minnaert (1933) and Strasberg (1956) verified the validity of Minnaert's analysis.

Following Minnaert, more attention was concentrated on gas bubble dynamics while cavitation dynamics seem to have received little attention. Smith (1935) investigated the behaviour of gas bubbles in an acoustic field. He derived an expression for the energy loss due to the radiation of acoustic waves by the bubble. Smith, however, mentioned that other sources of energy losses probably exist. The predictions

of Smith were later verified when Pfriem (1940) and Saneyosi (1941) investigated the thermal behaviour of pulsating gas bubbles in liquids. Their analysis showed that a non-uniform temperature field exists in the gas in the bubble. This causes a flow of heat inside the bubble which is conducted to the surrounding liquid. The above losses of energy by the bubble contribute to the damping of bubble pulsation in liquids. Poritsky (1951) investigated the effects of the surrounding liquid viscosity on the damping of bubble pulsation. A fuller treatment of the mathematics of cavitation and bubble dynamics is presented in Chapter 2.

Experimental work to determine the damping constants of bubble dynamics was carried out by several researchers. Different methods of measurement were used for this purpose.(Devin (1959)). Experimental work was carried out by Meyer and Tamm (1939), Cartensen and Foldy (1947), Lauer (1951), Exner (1951), Exner and Hampe (1953) and Haeske (1956). The investigators covered a wide range of bubble frequencies up to 300 KHz. Most of the experimental results were in good agreement with the theoretical analysis as presented by Devin (1959) and later by Chapman and Plesset (1971). Mayer and Tamm and Cartesen and Foldy reported results of the damping constant higher than the theoretical ones. Devin (1959) attributed that to specific aspects of their experiments. A common feature of the above experimental investigations is that the bubble was set into motion by external forces.

Free pulsation of gas bubbles in liquids was investigated by Strasberg(1954, 1956). Strasberg (1954) found that the non-sphericity of the bubble shape does not significantly change the resonance frequency of the bubble. Strasberg (1956) investigated the radiation of sound by pulsating gas bubbles in liquids and showed that the importance

of gas bubbles as sound radiators is associated with their volume pulsation rather than shape pulsation. The damping of free pulsating bubbles (Transient Pulsation) was investigated by Bauer (see Strasberg (1956) and Devin (1959)) and Koger and Houghton (1968). In both cases the bubbles were formed by compressing gas in submerged nozzles in water. The experimental results of free pulsating bubbles covered bubble frequencies up to 5 KHz. The reported experimental results of Bauer showed a damping constant of the pulsating bubble up to 25% higher than the theoretical values. The results of Koger and Houghton (1967) showed a wide scatter of the results of the damping constant. Koger and Houghton attributed the scatter of their data to the high experimental error of 15% estimated for their experiments. However, although the damping constant results scattered widely, the results of the resonance frequency fit well with the theoretical predictions of Minnaert. As will be shown later, the damping constant of a bubble pulsating near a scattering object (eg. a nozzle) can be greatly affected by the physical properties of the scattering object. The effects on the resonance frequency are very small (see discussion in Chapter 8). A fuller account of this phenomenon will be presented in detail in a later Chapter.

The study of cavitation dynamics was revived in the late forties and early fifties tackling such topics as cavitation due to the passage of sound waves in liquids, the condensation and evaporation of vapour bubbles and underwater explosions (Cole (1948), Plesset (1949) and Neppiras (1950), Neppiras and Noltingk (1951) and later by Houhgton (1963) and Hsieh (1965)). However, Robinson and Buchanan (1956) ivvestigated the undamped pulsation of a bubble. Their approach was based on the non-linear equation of cavitation dynamics rather than a simple energy balance followed by Minnaert (1963). The equation of

cavitation dynamics was linearised to account for small-amplitude bubble pulsation. They derived an expression for the resonance frequency of the bubble, taking into consideration the effects of the surface tension. By neglecting the surface tension effects, their expression reduces to that of Minnaert.

The discovery of the phenomenon of drag-reduction by polymer additives and the enthusiasm it received among many researchers led to a new wave of publications in the field of cavitation and bubble dynamics. The main aim of the researchers was to assess the effects of visco-elastic liquids on cavitation and its associated mechanical damages. It is still hoped that the behaviour of polymer solutions in non turbulent shear flow including cavitation and bubble dynamics, will present some useful information to explain the phenomenon of turbulent drag-reduction.

Cavitation dynamics was investigated theoretically and experimentally. Gas bubble dynamics was touched upon within the general frame of cavitation dynamics. Experimental results on the behaviour of bubble pulsation in visco-elastic liquids are scarce.

Cavitation dynamics in visco-elastic liquids were investigated theoretically by Fogler and Goddard (1970, 1971). The results of their calculations showed that the elasticity of the visco-elastic liquids significantly retarded the collapse of a void. The findings of Fogler and Goddard were confirmed later by Ting (1975). Tanasawa and Yang (1970) investigated the dynamic behaviour of gas bubbles in visco-elastic liquids. They found that the elasticity of the liquid reduced the effects of viscosity on the collapsed bubble. Yang and Lawson (1974) investigated bubble pulsation and cavitation in visco-elastic liquids. They found that the transient behaviour of the bubble in

visco-elastic liquids consisted of a decaying exponential component and a damped sinusoidal component. In purely viscous liquids, the transient behaviour is a damped sinusoidal component only.

Although Yang and Lawson (1974) did not investigate the effects of visco-elastic liquids on the damping constant of a free pulsating bubble, it will be shown later that the viscous component of the damping constant is significantly reduced in visco-elastic liquids. This conclusion is valid for visco-elastic liquids having relaxation time equal or greater than the time constant of the bubble. A point worth mentioning here is that the different investigators assumed the visco-elastic liquids to possess a viscosity which is equal to the shear viscosity. This assumption may not be valid for bubble pulsation in visco-elastic liquids. The velocity field of a pulsating bubble is irrotational and there exists the possibility that the visco-elastic liquids may exhibit high extensional viscosities under suitable conditions. Consequently, higher viscous losses are possible.

Experimental work on the effect of polymer solutions on cavitation were carried out by Ellis et al (1970) and Brennen (1970). The effect of polymer solutions was found to suppress the inception of cavitation. Gas bubble pulsation in polymer solutions was investigated by McComb and Ayyash (1976). Their aim was to find the effect of the polymer solutions on the damping of bubble pulsation. Polyox solutions were found to reduce the damping of bubble pulsation while the Separan AP-273 was more complex as it reduced and increased the damping in a concentration-dependence manner.

1.7 FREE OSCILLATION OF LIQUID COLUMN

Pulsating fluid flow is encountered in several fields in engineering, processing equipment and control devices. The physiologists have a special interest in the subject because the flow of blood is a pulsating one. Moreover, blood is a non-Newtonian fluid. The investigation of the behaviour of non-Newtonian fluids becomes thus important. In this section, we present a review of some of the works carried out on the oscillation of Newtonian and non-Newtonian liquids. A more detailed treatment of some aspects of the subject is presented in Chapter 4.

The easliest work on liquid column oscillation seems to be that of Menneret (1911). Menneret carried out experimental work to determine the damping constant of the decay of free liquid column oscillation. The experimental results of Menneret covered a wide range of Valensi number; a characteristic parameter of the oscillatory flow of liquid column. No theoretical work was developed at the time to correlate Menneret's data.

The theoretical work on liquid column oscillation was carried out with the view of deriving an expression for the damping constant of liquid column oscillation. The difficulty which the researchers encountered was the complex flowfield of the liquid column oscillation. Valensi (1947) investigated the damping of liquid column oscillation and derived an expression for the damping constant in terms of the dimensionless parameter known later as Valensi number. Valensi's expression failed to correlate Menneret's data. In a different approach to the problem, Valensi and Von Karman (1948) applied the boundary layer theory to obtain an asymptotic solution for small damping constants. Their calculations proved successful in describing the damping constant

at high Valensi numbers. Bird et al (1960) tackled the problem by assuming a fully developed laminar flow regime for the liquid column oscillation. They derived an expression for the damping constant which proved to be successful at low Valensi numbers (Biery (1969)).

Later researchers realised the inadequacy of the above solutions in describing the behaviour of liquid column oscillation. Ury (1962) and Biery (1963) resorted to numerical solutions of the equation of motion of liquid column oscillation. Their results were presented in the form of graphs. The solution of Ury gave higher values of the damping constant than Biery's at the same Valensi number.

Most of the experimental work on liquid column oscillation has been carried out in the past fifteen years. Ury (1962) presented the experimental results of the damping of a mercury column in a plastic tube. The results of the damping constant agreed with his analysis. Biery (1963) investigated several Newtonian liquids oscillating in a glass manometer. His results were in good agreement with his calculations. Park and Baird (1976) investigated the oscillation of liquid column in a glass tube. They showed that the results of the mercury column agreed with Ury's calculations while the results of the other Newtonian liquids agreed with the calculations of Biery. In Chapter 4 we present our view of the discrepancy between Ury's results and Biery's.

The oscillation of non-Newtonian visco-elastic liquid columns was investigated by Biery (1964). Biery used concentrated solutions of Polyox WSR-301. He found that the steady-state rheological models of the visco-elastic solutions can describe the unsteady behaviour of the solutions when the characteristic time of the liquid column oscillation is much greater than the relaxation time of the liquid. McComb (1974)

investigated the effect of drag-reducing additives on the total number of oscillations of liquid column in a flexible PVC tube. McComb's results showed that the addition of a few parts per million of Polyox WSR-301 to a water column more than doubled the number of oscillations of the column. Separan additives were found to have no efeect.

Ayyash and McComb (1976) investigated the effects of pure surfactants on the damping of liquid columns. Their results showed that the Aerosol additives increased the number of oscillations of a liquid column in a PVC tube. The effect of the Aerosol additives was much less than the Polyox WSR-301 additives as reported by McComb. In a glass tube both polymer additives and Aerosol additives failed to increase the total number of oscillations of the liquid column.

1.8 GENERAL ARRANGEMENT OF THE THESIS

In the following chapters of this thesis, we present the results of our investigation of the effects of drag-reducing polymer additives on oscillatory fluid flow. We look at some theoretical aspects of cavitation and bubble pulsation dynamics in Newtonian fluids. We then account for the effects of the non-Newtonian properties of the visco-elastic solutions on the dynamics of bubble pulsation.

Following this, we present the results of our investigation of the effects of adsorbed layers of polymer on turbulent shear flow and the damping of liquid column oscillation, together with the effects of polymer additives on liquid column oscillation.

In the rest of the chapters, the experimental aspects of the investigation of bubble dynamics are presented, followed by a discussion of the experimental results of our investigation.

CHAPTER 2

THEORETICAL ASPECTS OF BUBBLE DYNAMICS

2.1 INTRODUCTION

The study of bubble dynamics dates back to more than a century ago when Besant (1859) formulated the problem by considering an infinite mass of homogeneous incompressible fluid at rest and acted upon by no forces. A spherical portion of the fluid is suddenly annihilated. It is required to find the instantaneous alteration of pressure at any point of the mass, and the time in which the cavity will be filled up. The pressure at an infinite distance being supposed to remain constant.

Lord Rayleigh (1917) tackled the problem by assuming a zero pressure inside the cavity, and the fluid to be inviscid. Following a simple energy balance by equating the work done by the pressure at infinity to the kinetic energy acquired by the fluid, Lord Rayleigh obtained the following expression:

$$\dot{R}^2 = \frac{2P_\infty}{3\rho}\left(\frac{R_o^3}{R^3} - 1\right) \qquad (2.1)$$

where

R_o = Cavity radius at t = 0

R = Instantaneous cavity radius

$\dot{R}$ = Velocity of cavity wall

P_∞ = Pressure at infinity

ρ = Fluid density

The equation of motion of the cavity wall could then be derived straight forwardly from Equation (2.1). Differentiating Equation (2.1)

$$2\dot{R}\ddot{R}R^3 + 3R^2\dot{R}^3 = -\frac{2P_\infty}{\rho}R^2\dot{R} \qquad (2.2)$$

and then dividing both sides of Equation (2.2) by $2\dot{R}R$, there results

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{P_\infty}{\rho} \tag{2.3}$$

Equation (2.3) is the equation of motion of a cavity with zero inside pressure. In our analysis below, Equation (2.3) will be derived as a special case of the general equation of motion of a bubble pulsating in a viscous incompressible fluid.

Minnaert (1933), in studying the sound generated by running water, derived an expression for the resonance frequency of a bubble pulsating in an incompressible fluid. On the basis of simple energy balance, equating the potential energy of the bubble at its minimum volume to the maximum kinetic energy of the surrounding liquid mass, he derived the following expression for the resonance frequency of bubble oscillations

$$\omega_o = \frac{1}{R_o}\left(\frac{3\gamma P_\infty}{\rho}\right)^{\frac{1}{2}} \tag{2.4}$$

where

ω_o = Resonance frequency of the bubble in rad/sec

R_o = Equilibrium radius of the bubble

γ = Ratio of specific heats of the gas in the bubble: c_p/c_v

Minnaert assumed the gas in the bubble to undergo an adiabatic process, and that no energy losses are involved in the pulsation of the bubble. Minnaert's bubble is, in fact, a simple harmonic system analogous to a simple spring-mass system; the gas pressure in the bubble represents the stiffness of the spring and the surrounding liquid represents the inert mass.

Later researchers studied other aspects of bubble dynamics and in

particular the different mechanisms causing the damping of bubble pulsation. Smith (1935) derived an expression for the energy losses of a pulsating bubble due to acoustic radiation. Pfriem (1940) and Saneyosi (1941) investigated the thermal damping of pulsating bubbles while Poritsky (1951) investigated the viscous losses. Strasberg (1956) investigated the sound pressure waves associated with the pulsation of bubbles and showed that the significance of the pulsating bubbles as a source of sound in liquids is associated with their volume pulsation, rather than shape pulsation. Devin (1959) presented a coherent work analysing the different mechanisms of energy losses of pulsating bubbles.

Experimental work to determine the damping constants of pulsating bubbles accompanied the developments in the theory and covered a wide range of bubble sizes (Exner (1951), Exner and Hampe (1953), Haeske (1956)). Experimental results of the damping constants of pulsating bubbles were in good agreement with the theoretical analysis as presented by Devin (1959).

Below, we are going to derive the equation of motion of bubble dynamics. In our approach we will start from the basics of fluid dynamics, ie. the equation of continuity and the general equation of motion for a real fluid. Rayleigh's cavity and Minnaert's bubble will be derived in the course of our analysis as special cases of the general equation of bubble dynamics.

2.2 THE GENERAL EQUATION OF BUBBLE DYNAMICS

The equation of continuity in spherical coordinates (Pai 1956) is:

$$\frac{1}{r^2}\frac{\partial}{\partial r}(\rho U_r r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho U_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho U_\phi) = 0 \qquad (2.5)$$

while the equation of motion in the radial direction (Pai (1956)) is:

$$\rho\left[\frac{DU_r}{Dt} - \frac{U_\theta^2 + U_\phi^2}{r}\right] = F_o + \frac{1}{r^2\sin\theta}\left[\frac{\partial r^2\sin\theta\ \delta_{rr}}{\partial r} + \frac{\partial r\ \sin\theta\tau_{\theta r}}{\partial\theta} + \frac{\partial r\tau_{\phi r}}{\partial\phi}\right] - \frac{\delta_{\theta\theta} + \delta_{\phi\phi}}{r} \qquad (2.6)$$

where

u = velocity of the fluid

r = radial distance from the centre of the sphere

ρ = fluid density

F_o = Body forces per unit volume of the fluid

δ_{rr}, $\delta_{\theta\theta}$ $\delta_{\phi\phi}$ = normal stresses in r, θ and φ directions respectively

$\tau_{\theta r}$, $\tau_{\phi r}$ = shear stresses

subscripts

r, θ, φ = spherical coordinates

In an infinite mass of an incompressible viscous fluid there is a spherical gas-filled bubble. The centre of the bubble is the origin of the spherical coordinates. The bubble wall moves symmetrically and in a radial direction only. The static pressure at an infinite distance from the centre of the bubble is assumed to be constant.

Since the motion is assumed to be radial, all derivatives with respect to θ and/or φ vanish. Shear stress components vanish as well. Equation (2.6), in the absence of any body forces, becomes

$$\rho\frac{DU_r}{Dt} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta\ \delta_{rr})\right] - \frac{\delta_{\theta\theta} + \delta_{\phi\phi}}{r} \qquad (2.7)$$

The normal stress δ is composed of P, the static pressure, and σ the deviatoric stress tensor. The normal stresses in spherical terms are:

$$\delta_{rr} = -P_{rr} + \sigma_{rr} \tag{2.8a}$$

$$\delta_{\theta\theta} = -P_{\theta\theta} + \sigma_{\theta\theta} \tag{2.8b}$$

$$\delta_{\phi\phi} = -P_{\phi\phi} + \sigma_{\phi\phi} \tag{2.8c}$$

where

$P_{rr} = P_{\theta\theta} = P_{\phi\phi} = P$ the static pressure

$\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{\phi\phi}$ = Deviatoric stress tensors in r, θ and ϕ directions respectively.

By definition:

$$\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{\phi\phi} = 0 \tag{2.9};$$

Also, in spherical terms, the operator $\frac{D}{Dt}$ is:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U_r \frac{\partial}{\partial r} + \frac{U_\theta}{r} \frac{\partial}{\partial \theta} + \frac{U_\phi}{r \sin\theta} \frac{\partial}{\partial \phi} \tag{2.10}$$

Since the motion is assumed to be symmetrical and radial, Equation (2.10) becomes:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U_r \frac{\partial}{\partial r} \tag{2.11}$$

Substituting Equations (2.8a), (2.8b), (2.8c) and (2.11) in Equation (2.7) we obtain:

$$\rho \left[\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} \right] = \frac{1}{r^2 \sin\theta} \left[\frac{\partial (r^2 \sin\theta \, (-P + \sigma_{rr}))}{\partial r} \right] - \frac{-P + \sigma_{\theta\theta} - P + \sigma_{\phi\phi}}{r} \tag{2.12}$$

Expanding Equation (2.12)

$$\rho \left[\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} \right] = \frac{1}{r^2 \sin\theta} \left[r^2 \sin\theta \frac{\partial}{\partial r} (-P + \sigma_{rr}) + (-P + \sigma_{rr}) 2r \sin\theta \right] - \frac{-2P + \sigma_{\theta\theta} + \sigma_{\phi\phi}}{r} \tag{2.13}$$

$$\rho\left[\frac{\partial U_r}{\partial t} + U_r\frac{\partial U_r}{\partial r}\right] = \frac{\partial}{\partial r}(-P + \sigma_{rr}) + \frac{2(-P + \sigma_{rr})}{r} - \frac{-2P + \sigma_{rr} + \sigma_{rr}}{r} \tag{2.14}$$

Substituting Equation (2.9) in (2.14)

$$\rho\left[\frac{\partial U_r}{\partial t} + U_r\frac{\partial U_r}{\partial r}\right] = -\frac{\partial P}{\partial r} + \frac{\partial \sigma_{rr}}{\partial r} + \frac{3\sigma_{rr}}{r} \tag{2.15}$$

By symmetry and continuity

$$U_r = \frac{R^2\dot{R}}{r^2} \tag{2.16}$$

where

R = radius of the bubble

$\dot{R}$ = velocity of the bubble wall

r = radial distance from the centre of the bubble, $r > R$.

Substituting Equation (2.16) in (2.15):

$$\rho\left[\frac{\partial}{\partial t}\left(\frac{R^2R'}{r^2}\right) + \frac{1}{2}\frac{\partial}{\partial r}\left(\frac{R^4\dot{R}^2}{r^4}\right)\right] = \frac{-\partial P}{\partial r} + \frac{\partial \sigma_{rr}}{\partial r} + \frac{3\sigma_{rr}}{r} \tag{2.17}$$

Expanding Equation (2.17)

$$\rho\left[\frac{2R\dot{R}^2}{r^2} + \frac{R^2R''}{r^2} + \frac{1}{2}\frac{\partial}{\partial r}\left(\frac{R^4\dot{R}^2}{r^4}\right)\right] = -\frac{\partial P}{\partial r} + \frac{\partial \sigma_{rr}}{\partial r} + \frac{3\sigma_{rr}}{r} \tag{2.18}$$

Integrating both sides of Equation (2.18) from $r = R$ to $r = \infty$

$$\rho\left[\int_R^\infty \frac{2R\dot{R}^2}{r^2}\,dr + \int_R^\infty \frac{R^2R''}{r^2}\,dr + \frac{1}{2}\int_R^\infty \frac{\partial}{\partial r}\left(\frac{R^4\dot{R}^2}{r^4}\right)dr\right] =$$

$$\int_R^\infty -\frac{\partial P}{\partial r}\,dr + \int_R^\infty \frac{\partial \sigma_{rr}}{\partial r}\,dr + 3\int_R^\infty \frac{\sigma_{rr}}{r}\,dr \tag{2.19}$$

$$\rho\left[\frac{-2R\dot{R}^2}{r}\Big|_R^\infty - \frac{R^2\ddot{R}}{r}\Big|_R^\infty + \frac{1}{2}\frac{R^4\dot{R}^2}{r^4}\Big|_R^\infty\right]$$

$$= -P\Big|_R^\infty + \sigma_{rr}\Big|_R^\infty + 3\int_R^\infty \frac{\sigma_{rr}}{r}\,dr \qquad (2.20)$$

At infinity, the boundary conditions are

$$U_r = 0$$

$$P = P_\infty$$

$$\sigma_{rr} = 0$$

Equation (2.20) becomes

$$\rho\left[R\ddot{R} + \frac{3}{2}\dot{R}^2\right] = P_R - P_\infty - \sigma_{rr_R} + 3\int_R^\infty \frac{\sigma_{rr}}{r}\,dr \qquad (2.21)$$

where P_R is the pressure at the surface of the bubble just within the liquid.

Equation (2.21) is the general equation of motion of a cavity in an incompressible real fluid. In the case of a cavity with zero inside pressure in an inviscid fluid (Rayleigh's cavity)

$$P_R = 0 \qquad (2.22a)$$

$$\sigma_{rr} = 0 \qquad (2.22b)$$

Substituting (2.22a) and (2.22b) in (2.21) we obtain

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = -\frac{P_\infty}{\rho} \qquad (2.23)$$

Equation (2.23) is the same as (2.3) derived by Rayleigh.

If the bubble described in Equation (2.21) is a gas-filled one which pulsates around an equilibrium radius R_o, then for adiabatic pulsation

$$\frac{P_g}{P_o} = \left(\frac{R_o}{R}\right)^{3\gamma} \tag{2.24}$$

where

P_g = gas pressure inside the bubble

P_o = equilibrium pressure in the bubble

R_o = equilibrium bubble radius.

Following Plesset (1949) the equilibrium gas pressure is expressed as

$$P_o = P_\infty + \frac{2S}{R_o} \tag{2.25}$$

where

S = surface tension.

The continuity of normal stress at the bubble surface requires that

$$P_g + \sigma_{rrg}\Big|_R = P_R - \sigma_{rr\ell}\Big|_R + \frac{2S}{R} \tag{2.26}$$

where σ_{rrg} = radial deviatoric stress tensor in the gas. However, σ_{rrg} is very small and may be neglected from the start.

Subsituting Equations (2.24), (2.25) and (2.26) in Equation (2.21) we obtain

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \left(\frac{P_\infty}{\rho} + \frac{2S}{\rho R_o}\right)\left(\frac{R_o}{R}\right)^{3\gamma} - \frac{P_\infty}{\rho} - \frac{2S}{\rho R} + \frac{3}{\rho}\int_R^\infty \frac{\sigma_{rr}}{r}\,dr \tag{2.27}$$

For a Newtonian incompressible fluid

$$\sigma_{rr} = 2\mu e_{rr} \tag{2.28}$$

where

μ = dynamic viscosity

$e_{rr} = \frac{\partial u_r}{\partial r}$ = rate of strain.

Subsituting Equations (2.16) and (2.28) in (2.27) gives

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \left(\frac{P_\infty}{\rho} + \frac{2S}{\rho R_o}\right)\left(\frac{R_o}{R}\right)^{3\gamma} - \frac{P_\infty}{\rho} - \frac{2S}{\rho R} - \frac{4\mu\dot{R}}{\rho R} \qquad (2.29)$$

Equations (2.29) is the same as that derived by Houghton (1963). The surface tension of water is 73 dyne/cm while the atmospheric pressure is 10^6 dyne/cm^2. For bubbles of 0.1 cm in radius or larger, surface tension effects are small in comparison with static pressure. For bubbles of very small radii surface tension effects become larger and should be included in the equation of motion of bubble dynamics. In our analysis, the bubbles of interest are relatively large (R_o > 0.2 cm) and the surface tension effects would then be negligible and may be neglected.

Neglecting the surface tension effects and expressing Equation (2.27) in terms of P_g we obtain

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_g}{\rho} - \frac{P_\infty}{\rho} + \frac{3}{\rho}\int_R^\infty \frac{\sigma_{rr}}{r}\,dr \qquad (2.30)$$

Equation (2.30) is the general equation of motion of a gas-filled cavity pulsating in an incompressible real fluid. It is clear that Equation (2.30) is a non-linear differential equation. To describe the dynamics of small amplitude pulsation we will linearise Equation (2.30).

2.3 LINEARISATION OF THE EQUATION OF MOTION OF BUBBLE DYNAMICS

If the bubble described by Equation (2.30) pulsates in such a way that the pulsation amplitude is very small in comparison with the equilibrium radius of the bubble, then it is satisfactory to write

$$R = R_o + \Delta R \qquad (2.31)$$

where

$$\Delta R << R_o$$

Substituting Equation (2.31) in Equation (2.30) and neglecting terms of $(\Delta R)^2$ and higher we obtain

$$R_o \ddot{\Delta R} = \frac{P_g - P_\infty}{\rho} + \frac{3}{\rho} \int_R^\infty \frac{\sigma_{rr}}{r} \, dr \qquad (2.32)$$

The temperature field inside free pulsating gas bubbles is non-uniform. Such non-uniformity causes a heat flow from the bubble to the surrounding liquid. The thermal behaviour of the gas in the bubble would then no longer be an adiabatic one as assumed by Minnaert (1933). The polytropic exponent of the thermal process has a value $1 < k < 1.4$. In such a case the gas pressure P_g is expressed in terms of the equilibrium pressure as follows

$$P_g = P_o \left(\frac{R}{R_o}\right)^{3k} \qquad (2.33)$$

Equation (2.33) can be approximated by the first two terms of a series expansion

$$P_g = P_o - \frac{3k\Delta R P_o}{R_o} \qquad (2.34)$$

Substituting Equation (2.34) in Equation (2.32) we obtain

$$R_o \ddot{\Delta R} - \frac{3}{\rho} \int_{R_o}^\infty \frac{\sigma_{rr}}{r} \, dr + \frac{3kP_o}{\rho R_o} \Delta R = 0 \qquad (2.35)$$

Introducing v, $\dot{v}$, $\ddot{v}$ as the change in the volume of the pulsating bubble and its first and second derivatives respectively we obtain

$$v = 4\pi R_o^2 \Delta R \qquad (2.36a)$$

$$\dot{v} = 4\pi R_o^2 \dot{\Delta R} \qquad (2.36b)$$

$$\ddot{v} = 4\pi R_o^2 \ddot{\Delta R} \qquad (2.36c)$$

Substituting Equations (2.36a) and (2.36b) in Equation (2.35), there results

$$\frac{\rho}{4\pi R_o} \ddot{v} - \frac{3}{\rho} \int_{R_o}^{\infty} \frac{\sigma_{rr}}{r} dr + \frac{kP_o}{V_o} v = 0 \qquad (2.37)$$

For a Newtonian incompressible fluid

$$\sigma_{rr} = 2 \mu e_{rr}$$

and

$$e_{rr} = \frac{\partial U_r}{\partial r} = \frac{\partial}{\partial r} \left(\frac{R^2 R'}{r^2}\right) \qquad (2.38)$$

Linearisation of e_{rr} by introducing $R = R_o + \Delta R$ gives

$$e_{rr} = \frac{\partial}{\partial r} \left(\frac{R_o^2 \dot{\Delta R}}{r^2}\right)$$

or

$$e_{rr} = \frac{\partial}{\partial r} \left(\frac{\dot{v}}{4\pi r^2}\right)$$

solving the differentiation

$$e_{rr} = \frac{-\dot{v}}{2\pi r^3} \qquad (2.39)$$

Equation (2.39) in Equation (2.38) gives

$$\sigma_{rr} = \frac{-\mu \dot{v}}{\pi r^3} \qquad (2.40)$$

Substituting Equation (2.40) in Equation (2.37) gives

$$\frac{\rho \ddot{v}}{4\pi R_o} + 3 \int_{R_o}^{\infty} \frac{\mu \dot{v}}{4r^4} dr + \frac{kP_o}{V_o} v = 0$$

$$\frac{\rho}{4\pi R_o} \ddot{v} + \frac{\mu}{\pi R_o^3} \dot{v} + \frac{kP_o}{V_o} v = 0 \qquad (2.41)$$

Equation (2.41) is the general equation of motion of small amplitude bubble pulsation in a Newtonian incompressible fluid and is

identical to the equation derived by Devin (1959). Equation (2.41) describes a damped oscillatory system. The term $\frac{\rho}{4\pi R_o}$ represents the mass set into vibration, $\frac{\mu}{\pi R_o^3}$ represents the viscous losses and $\frac{kP_o}{V_o}$ the stiffness of the system.

If the viscous losses are very small and could be neglected, Equation (2.41) is reduced to

$$\frac{\rho}{4\pi R_o} \ddot{v} + \frac{kP_o}{V_o} v = 0 \qquad (2.42)$$

For an adiabatic pulsation, the polytropic exponent k is equal to the adiabatic exponent. The resonance frequency of the system would then be

$$\omega_o = \frac{1}{R_o} \left[\frac{3\gamma P_o}{\rho}\right]^{\frac{1}{2}} \qquad (2.43)$$

Equation (2.43) is that of the resonance frequency of Minnaert's bubble Equation (2.4).

2.4 PULSATING GAS BUBBLES AS A SOURCE OF SOUND IN LIQUIDS

The linearised equation of bubble dynamics in incompressible viscous fluids, derived in the preceeding section, states that the bubble behaves like a simple oscillatory system with viscous losses. As a result of such oscillations, the bubble radiates sound waves.

The sound of air bubbles pulsating in liquids has been investigated by Minnaert (1933), Meyer & Tam (1939) and Strasberg (1956). In this section, we will investigate the importance of bubbles as sound radiators, and will be following the analysis of Strasberg (1956).

Bubbles pulsating in liquids experience changes in both their volume and shape. Sound radiation is associated with both types of

pulsation. To find out a quantitative relationship between the pressures of both types of sound waves, the pulsation of the bubble wall is represented by a sum of surface harmonics (Lamb (1959)).

The instantaneous radius at the bubble surface at any time is

$$r(t, \theta, \phi) = R_o + \sum_n \Delta R_n S_n(\theta, \phi) \exp(2\pi i f_n t) \qquad (2.44)$$

where

R_o = equilibrium radius

S_n = surface harmonic of order n

ΔR_n = amplitude of oscillation associated with the nth order

f_n = frequency of oscillation.

A bubble pulsating in a mode corresponding to n = 0 experiences changes in volume but no change in shape. For n = 1, the bubble pulsation corresponds to a translational oscillation of the bubble without changes in either the shape or the volume. At higher modes the bubble changes shape while the volume is fixed.

Each mode of pulsation has its own natural frequency. For the simple volume pulsation (n = 0) the natural frequency (Minnaert (1933)) is

$$f_o = \frac{1}{2\pi R_o} \left[\frac{3\gamma P_o}{\rho}\right]^{\frac{1}{2}} \qquad (2.45)$$

=

For n = 1, there is no natural frequency since translational pulsations have no restoring force. For n > 1, the natural frequency (Lamb (1945)) is

$$f_n = \left[(n^2 - 1)(n + 2) \frac{S}{\rho R_o}\right]^{\frac{1}{2}} \Big/ 2\pi R_o \qquad (2.46)$$

where S is the surface tension.

The pressure amplitude of the sound wave radiated by simple

volume pulsation (n = o) is

$$P_{ac} = \frac{3\gamma P_o \Delta R}{d} \qquad (2.47)$$

where d is the radial distance from the centre of the bubble. For higher modes, the pressure amplitude at a distance much smaller than a wave length is

$$P = (n - 1)(n + 2)\left(\frac{S}{R_o}\right)\left(\frac{\Delta R_n}{d}\right)\left(\frac{R_o}{d}\right)^n S_n \qquad (2.48)$$

Strasberg (1956) calculated the sound pressure and natural frequency for bubbles, 0.33 and 0.033 cm in radius, pulsating in water. According to his calculations, the ratio of the pressure amplitude of sound waves emitted by shape pulsation to the pressure amplitude of sound waves emitted by volume pulsation ranged between 2.4×10^{-7} - 1.15×10^{-12} indicating that sound pressure associated with shape pulsation are indeed negligible in comparison with those of the volume pulsation.

Accordingly we will deal with the bubbles as sound radiators in their volume pulsation mode only.

2.5 MECHANISMS OF ENERGY LOSSES BY PULSATING GAS BUBBLES IN LIQUIDS

Lord Rayleigh's analysis of cavitation was based on the assumption that the fluid surrounding the cavity is an inviscid one. Such an assumption eliminates the possibility of viscous losses. Minnaert, on the other hand, assumed an adiabatic behaviour for the gas in the bubble thus eliminating any heat transfer. His analysis was based on the assumption of no energy losses by the pulsating bubble.

We have already shown above (Equation (2.41)) that bubbles

pulsating in Newtonian fluids experience energy losses in the form of viscous losses. In section (2.4) we investigated the bubbles as sound radiators. Being sound radiators the bubbles expend a part of their energy in radiating sound waves because of the acoustic impedance of the surrounding medium. Moreover, the temperature of the gas in the bubble changes as a result of the pulsation leading to a temperature difference inside the bubble thus causing a transfer of heat to the surrounding liquid. This transfer of heat is another form of energy loss.

Energy losses by pulsating bubbles are well documented in the literature (Devin (1959), Chapman and Plesset (1971) and Prosperetti (1977)). In this section we will derive the appropriate expressions for the different forms of energy losses.

2.5.1 Viscous Losses

In section (2.2) we derived the general equation of bubble dynamics in a real fluid. The equation was then linearised to account for small amplitude bubble pulsation. The general equation of small amplitude bubble pulsation in a real fluid is

$$\frac{\rho}{4\pi R_o}\ddot{v} - 3\int_{R_o}^{\infty}\frac{\sigma_{rr}}{r}\,dr + \frac{kP_o}{V_o}v = 0 \qquad (2.49)$$

For a Newtonian liquid the equation is reduced to

$$\frac{\rho}{4\pi R_o}\ddot{v} + \frac{\mu}{\pi R_o^3}\dot{v} + \frac{kP_o}{V_o}v = 0 \qquad (2.50)$$

Equation (2.50) is a second-order linear differential equation describing a damped oscillatory system with viscous energy losses. The expression $\frac{\mu}{\pi R_o^3}$ represents the viscous damping factor.

The viscous losses are directly proportional to the viscosity of the liquid surrounding the bubble. The larger the viscosity the larger the viscous losses. Viscous losses are inversely proportional to the cube of the bubble radius. The smaller the bubble radius the larger the viscous losses. In water, viscous losses are important only for bubbles of .01 cm in radius or smaller. For larger bubbles they are very small in comparison with the acoustic or thermal energy losses.

2.5.2 Acoustic Radiation Losses

In section (2.4) bubbles were investigated as sound radiators in liquids. From an acoustic point of view bubbles are considered as simple sources; the radius of the source is very small in comparison with the radiated sound wave length. Because of the acoustic impedance of the surrounding medium bubbles expend a part of their energy in radiating sound waves.

The acoustic impedance is a complex whose real part is the acoustic radiation resistance; the component associated with the dissipation of energy. The imaginary part of the acoustic impedance is the acoustic reactance which results from the effective mass of the medium (Kinsler and Frey (1962)).

The acoustic impedance is expressed as follows

$$Z = \rho c \frac{k^2 r^2}{1 + k^2 r^2} + j \rho c \frac{kr}{1 + k^2 r^2} \qquad (2.51)$$

where

ρ = liquid density

c = velocity of sound in the liquid

r = radial distance

A freely pulsating bubble in a liquid, eg. a bubble formed at a nozzle, pulsates under the influence of an initial velocity of the bubble wall rather than an initial displacement (Houghton (1964)). The velocity of the bubble wall

$$\dot{v} = \dot{V}_o \exp j\omega t \tag{2.55}$$

The velocity potential of a pulsating simple source in a compressible fluid

$$\phi = \frac{\dot{V}_o}{4\pi r} \exp\left[j\omega(t - r/c)\right] \tag{2.56}$$

We introduce the compressibility here because it is the source of acoustic dissipation. In a truely incompressible medium, the velocity of sound is infinity, consequently no acoustic dissipation will arise. The introduction of compressibility in this section contradicts our assumption in section (2.2) that the fluid medium surrounding the bubble is incompressible. However, due to the very low compressibility of liquids it is usually assumed that liquids are incompressible. In dealing with acoustics, liquids can no longer be taken as incompressible. As will be shown below, the introduction of compressibility will not affect the inertia term of the pulsating system.

The acoustic pressure due to a simple source is

$$P_{ac} = \frac{\rho \partial \phi}{\partial t} \tag{2.57}$$

$$P_{ac} = \frac{\rho j\omega \dot{V}_o}{4\pi r} \exp\left[j\omega(t - r/c)\right]$$

At the surface of the bubble

$$P_{ac} = \frac{\rho j\omega \dot{V}_o}{4\pi R_o} \exp\left[j\omega(t - \frac{R_o}{C_o})\right] \tag{2.58}$$

$k = \omega/c$

ω = frequency radians/sec

For $k^2r^2 << 1$, ie. small air bubbles in water, the radiation resistance becomes

$$R_{ac} = \rho c\, k^2 r^2 \tag{2.52}$$

On the surface of the bubble

$$R_{ac} = \frac{\rho\, \omega^2 R_o^2}{c} \tag{2.53}$$

Smith (1935), in investigating the effect of the scattering of sound waves by the bubble, introduced Equation (2.53) as an energy dissipative term in Minnaert's bubble. The equation of the damped pulsating bubble derived by Smith is

$$\frac{\rho}{4\pi R_o}\ddot{v} + \frac{\rho\omega^2}{4\pi c}\dot{v} + 3\frac{\gamma P_o}{V_o}v = 0 \tag{2.54}$$

The damping of a bubble pulsation due to acoustic radiation was later investigated by Yosioka et al (1955), Devin (1959), Barnes and Anderson (1968), Chapman and Plesset (1971) and Prosperetti (1977).

The above cited works, with the exception of Chapman and Plesset, investigated the behaviour of the bubbles under the influence of external driving forces. Chapman and Plesset investigated the behaviour of a freely pulsating bubble and derived an expression for the energy losses as a whole (Thermal, Acoustic radiation and viscous). Below we will derive an expression for the acoustic radiation losses of a freely pulsating bubble.

or by expanding exp $(j\omega\frac{c}{R_o})$ to first order of ω

$$P_{ac} = \frac{\rho j\omega V_o}{4\pi R_o} \left(1 - \frac{j\omega R_o}{c}\right) \exp(j\omega t) \qquad (2.59)$$

$$P_{ac} = \frac{\rho \dot{v}}{4\pi R_o} + \frac{\rho\omega^2}{4\pi c} v \qquad (2.60)$$

For a free pulsating bubble the acoustic pressure at the surface of the bubble is simply the difference between the pressure inside the bubble and the ambient pressure. Equations (2.34) and (2.60) together give

$$\frac{\rho}{4\pi R_o}\ddot{v} + \frac{\rho\omega^2}{4\pi c}\dot{v} + \frac{3kP_o}{V_o} v = 0 \qquad (2.61)$$

Equation (2.61) is the same as that derived by Devin (1959) for a bubble pulsating under the influence of an external driving force, with the right hand side of the equation set to zero.

2.5.3 Thermal Losses

A pulsating bubble in a liquid changes volume. The change in the volume of the gas in the bubble is associated with changes in the other properties of the gas, eg. pressure, density and temperature. When Minnaert investigated the pulsation of bubbles in liquids he assumed an adiabatic process for the expansion-contraction of the gas. In an adiabatic process there is no transfer of heat. Had Minnaert assumed an isothermal behaviour of the gas he would have derived an expression for the frequency of the bubble that is $\gamma^{-\frac{1}{2}}$ less than the expression he derived assuming an adiabatic behaviour.

The thermal damping of a pulsating bubble was first investigated by Pfriem (1940) and then by Saneyosi (1941). Devin (1959)

followed the derivation of Pfriem and derived the thermal damping constant of bubbles in water. Devin's thermal damping constant has the form

$$\delta th = 4.41 \times 10^{-4} f_o^{\frac{1}{2}} \qquad (2.62)$$

The thermal damping constant derived by Devin has been used by us in comparing our experimental results of the damping constant of bubbles with the theoretical predictions.

Plesset and Hsieh (1960) have investigated the behaviour of gas bubbles in a liquid with an oscillating pressure field. According to their analysis the bubble behaves isothermally at low frequency of the oscillating pressure field, and it behaves adiabatically at high frequencies. Such a conclusion is based on the assumption that the bubble interior is uniform, ie. the properties of the gas in the bubble are a function of time only. When allowing for non-uniformities in the bubble interior, ie. when the properties of the gas in the bubble become a function of time and space the bubble behaves isothermally at the high frequencies.

Prosperetti (1977) investigated the same problem and reached the same conclusion as Plesset and Hsieh (1960). According to Prosperetti, the bubble behaves isothermally at low frequencies because the thermal energy associated with the compression and expansion of the bubble is transferred by the liquid surrounding the bubble at nearly the same rate at which it is produced. At high frequencies, when the pressure wave length is much smaller than the bubble diameter and the number of wave lengths contained in the bubble becomes large, the behaviour is

isothermal because net internal energy variations in the bubble are small.

Chapman and Plesset (1971) investigated the thermal behaviour of a free pulsating bubble in water. In their analysis they deduced expressions for the viscous, acoustic radiation and thermal damping constants of the bubble. Their results were consistent with those of Devin (1959).

In this section, we will not involve ourselves deeply in the mathematical analysis of the thermodynamic behaviour of the gas in the bubble.

The thermal damping of a pulsating bubble is essentially controlled by the thermodynamic behaviour of the gas in the bubbles. The physical properties of the bubble and the gas and the change in these properties are of prime importance in the thermal damping of the bubble. The thermal properties of the surrounding liquid are of little importance because of the wide difference between the values of the thermal properties of liquids and gases, eg. specific heat and thermal conductivity. The thermal properties of the surrounding liquid would become important if they could be changed by orders of magnitude. In this research, the only change in the thermal properties of the surrounding liquid would happen due to the polymer additives. Although the polymer additives change the thermal properties of water (Bellet et al (1975)) the change is not great enough to render the thermal properties of the polymer solutions important in determining the thermal damping of bubbles. The thermal damping of a pulsating bubble would still be controlled by the thermal behaviour of the gas in the bubble, even in polymer solutions.

Instead we are going, in this section, to show how the thermal losses are introduced into the equation of motion of a pulsating bubble. Also, we are going to outline the major equations and boundary conditions involved in the thermal behaviour of the bubble. The viscous and acoustic radiation losses will appear in the course of the analysis.

The continuity of pressure at the gas-liquid interface means that the pressure at the bubble surface just within the liquid equals the pressure at the bubble surface just within the gas. The pressure on the gas side of the interface is the sum of the ambient pressure and the change in pressure associated with the change of the bubble volume taking the thermal losses into consideration. The pressure on the liquid side of the interface is the sum of the ambient pressure and the acoustic pressure including the viscous and acoustic radiation losses.

The pressure on the liquid side

$$P_r = P_o + P_{ac} + \sigma_{rr} \tag{2.63}$$

P_{ac} is defined by Equation (2.60)

$$\sigma_{rr} = \frac{\mu}{\pi R_o^3} \dot{v}$$

The pressure on the gas side

$$P_g = P_o + \Delta P$$

where ΔP is the change in pressure associated with the change in bubble volume. According to Equation (2.34) (assuming no thermal losses)

$$P_g = P_o - \frac{3kP_o}{V_o} v$$

when the thermal losses are taken into consideration, a thermal energy dissipative term is introduced. The term has the form $(\mu_{th}\dot{v})$ where μ_{th} is assumed to be the thermal damping factor of the pulsating bubble. Equation (2.34) thus becomes

$$P_g = P_o - \frac{3kP_o}{V_o} v - \mu_{th}\dot{v} \qquad (2.64)$$

Equation (2.58), (2.63) and (2.64) give

$$\frac{\rho}{4\pi R_o}\ddot{v} + \left(\frac{\mu}{\pi R_o^3} + \frac{\rho\omega^2}{4\pi c} + \mu_{th}\right)\dot{v} + \frac{3kP_o}{V_o}v = 0 \qquad (2.65)$$

Equation (2.65) is the linearised equation of motion of a bubble pulsating in a Newtonian liquid. The equation describes a damped harmonic oscillation.

The actual change in the gas pressure associated with the change in the bubble volume is found by assuming that the change in the physical properties of the gas and the temperature in the liquid to be small:

$$\rho_g = \rho_{og}(1 + h)$$

$$T_g = T_{og}(1 + \theta_g)$$

$$T_\ell = T_{o\ell}(1 + \theta_\ell)$$

For a perfect gas P_g becomes

$$P_g = P_o(1 + h + \theta) \qquad (2.66)$$

Equation (2.66) is solved with the aid of the equations of continuity, momentum and conservation of energy in the gas phase

$$\left(\frac{\partial h}{\partial t}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 U_r) = 0 \qquad \text{Equation of Continuity}$$

$$\frac{\partial U_r}{\partial t} + \frac{P_o}{\rho_g}\frac{\partial P}{\partial r} = 0 \qquad \text{Equation of Momentum}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta_g}{\partial r}\right) + \frac{P_o}{K_g T_\infty}\frac{\partial h}{\partial t} = \frac{1}{D_g}\frac{\partial \theta_g}{\partial t} \qquad \text{Equation of Conservation of Energy}$$

The subscripts g and ℓ refer to gas and liquid respectively.

$D = k/\rho c_v$ the thermal diffusivity

The thermal damping factor μ_{th} is the function of a complex quantity G. The complex quantity G depends essentially on two dimensionless parameters (Chapman and Plesset (1971), Prosperetti (1977)). The two parameters are

$$G_1 = \left[\frac{(D_g/\omega)^{\frac{1}{2}}}{C_g/\omega}\right]^2$$

and

$$G_2 = \frac{\omega R_o^2}{D_g}$$

The first parameter is simply the square of the ratio between the thermal penetration depth and the sound wave length. This parameter would become important only at very high frequencies (higher than 3×10^9 c/s, Plesset (1964)). The second parameter is simply the square of the ratio between the bubble radius and the thermal penetration depth. In the bubble of interest in this work, the first parameter is very small and could be neglected while the second parameter is the essential one. Consequently, the thermal energy losses of a pulsating bubble are, in general, essentially controlled by the physical parameters of the bubble and the gas. The effects of the surrounding liquid are negligible.

CHAPTER 3

A THEORETICAL ASSESSMENT OF THE EFFECT OF NON-NEWTONIAN TERMS IN THE EQUATION OF MOTION OF BUBBLE DYNAMICS

3.1 In Chapter 2 we discussed some theoretical aspects of bubble dynamics. A general equation of bubble dynamics in a real fluid was derived. The equation was linearised to describe small-amplitude bubble pulsation. The linearised general equation of bubble dynamics in a general fluid, taking into consideration the acoustic and thermal energy losses, has the form

$$\frac{\rho}{4\pi R_o}\ddot{v} + c_1\dot{v} - 3\int_{R_o}^{\infty}\frac{\sigma_{rr}}{r}\,dr + \frac{P_o k}{V_o}v = 0 \qquad (3.1)$$

where c_1 is a term representing the thermal and acoustic radiation damping factor. The notation is that of Chapter 2.

Equation (3.1) was solved for the case of a Newtonian liquid. In Newtonian liquids, the deviatoric stress tensor σ_{rr} and the strain rate tensor e_{ij} are related together through the equation

$$\sigma_{ij} = 2\mu e_{ij} \qquad (3.2)$$

The Newtonian viscosity, μ, depends on the temperature only and is independent of the strain rate.

In this Chapter, we will discuss the behaviour of pulsating bubbles in non-Newtonian liquids. Non-Newtonian liquids are those liquids whose viscosity is not a function of temperature only but of other factors such as the strain rate and the past history of the fluid.

Non-Newtonian liquids may be classified as:

a) Non-Newtonian viscous liquids such as Bingham plastics and power-law liquids.

b) Non-Newtonian visco-elastic fluids such as polymer melts and polymer solutions.

Although the relationship between stress and rate of strain in both types of non-Newtonian liquids is a non-linear one, the non-Newtonian visco-elastic liquids exhibit elastic effects in addition to viscous effects while the non-Newtonian viscous liquids exhibit viscous effects only. Our concern, in this Chapter, is the dynamic behaviour of pulsating bubbles in visco-elastic liquids.

Bubble dynamics in non-Newtonian viscous and visco-elastic liquids has been investigated by several researchers. Their main purpose was to investigate the effect of the non-Newtonian terms on the growth and collapse of cavitation bubbles. Bubble dynamics in non-Newtonian viscous liquids were investigated by Yang and Yeh (1966) and Shima and Tsujino (1976) who presented numerical solutions only.

In visco-elastic liquids, the theoretical and experimental study of bubble dynamics went hand-in-hand. Fogler and Goddard (1970) discussed the collapse of spherical cavities in visco-elastic liquids and presented some numerical solutions of the equation of cavity dynamics. In comparison with viscous liquids, the visco-elastic liquids have the effect of retarding the collapse of a void. Tanasawa and Yang (1970) presented numerical solutions for the general equation of bubble dynamics in an Oldroyd visco-elastic liquid. Their calculations disclosed that the viscous damping effects on the bubble collapse are less in visco-elastic liquid than in pure viscous liquids. They also found that, in a visco-elastic liquid, a bubble collapses faster under

adiabatic conditions than isothermal conditions. Yang and Lawson (1974, 1974) investigated free and forced pulsation of bubbles in visco-elastic liquids in connection with the stability of bubbles and the onset of incipient cavitation. Ting (1975) discussed the visco-elastic effects of polymer solutions on bubble dynamics and showed that the collapse of cavities in polymer solutions is retarded in comparison with that in Newtonian and inviscid liquids.

Experimental work to investigate the behaviour of bubbles in visco-elastic liquids has been mainly confined to cavitation bubbles (Ellis et al (1970), Brennen (1972)). Small amplitude bubble pulsation in visco-elastic liquids has received very little attention in the experimental field. The paper by McComb and Ayyash (1976) is probably the only published work in the field.

3.2 SMALL AMPLITUDE BUBBLE PULSATION IN VISCO-ELASTIC LIQUIDS

The flow behaviour of visco-elastic liquids is quite complicated. There is no single model that would describe the dynamic behaviour of these liquids. Instead, several models have been proposed to account for the different behaviour patterns of visco-elastic liquids (Wilkinson (1960), Bland (1960) and Darby (1976)).

In our approach to investigate the dynamic behaviour of pulsating bubbles in visco-elastic liquids, we will use an Oldroyd model to describe the behaviour of the visco-elastic liquid. The Oldroyd model has the expression

$$(1 + t_1 D)\sigma_{rr} = 2\mu\,(1 + t_2 D)e_{rr} \qquad (3.3)$$

where, after linearisation (Lumley (1971))

$$D \equiv \frac{\partial}{\partial t}$$

t_1 = stress relaxation time

t_2 = strain rate relaxation time (retardation time)

Equations (3.3) and (3.1) give

$$\frac{\rho v}{4\pi R_o}\ddot{v} + c_1\dot{v} - 6\mu \int_{R_o}^{\infty} \frac{(1 + t_2 D)}{(1 + t_1 D)} \frac{e_{rr}}{r}\, dr + \frac{P_o k}{V_o} v = 0 \qquad (3.4)$$

or

$$\ddot{v} + \frac{4\pi R_o c_1}{\rho}\dot{v} - \frac{24\pi\mu R_o}{\rho} \int_{R_o}^{\infty} \frac{(1 + t_2 D)}{(1 + t_1 D)} \frac{e_{rr}}{r}\, dr + \omega^2 v = 0 \qquad (3.5)$$

where ω is the angular frequency of the bubble.

In linearised form (see Chapter 2)

$$e_{rr} = \frac{-\dot{v}}{2\pi r^3} \qquad (3.6)$$

Equations (3.5) and (3.6) give

$$\ddot{v} + \frac{4\pi R_o c_1}{\rho}\dot{v} + \frac{12\pi R_o \mu}{\rho} \int_{R_o}^{\infty} \frac{(1 + t_2 D)}{(1 + t_1 D)} \frac{\dot{v}}{\pi r^4}\, dr + \omega^2 v = 0 \qquad (3.7)$$

After integration we obtain

$$\ddot{v} + \frac{4\pi R_o c_1}{\rho}\dot{v} + \frac{4\mu}{\rho R_o^3} \frac{(1 + t_2 D)}{(1 + t_1 D)}\dot{v} + \omega^2 v = 0 \qquad (3.8)$$

Multiply Equation (3.8) by $(1 + t_1 D)$

$$(1 + t_1 D)\ddot{v} + (1 + t_1 D)\frac{4\pi R_o c_1}{\rho}\dot{v} + \frac{4\mu}{\rho R_o^3}(1 + t_2 D)\dot{v} + (1 + t_1 D)\omega^2 v = 0 \qquad (3.9)$$

Replacing D by $\frac{\partial}{\partial t}$ operating to the right and re-arranging the terms, we obtain

$$t_1\dddot{v} + (1 + \frac{4\pi R_o c_1}{\rho} t_1 + \frac{4\mu}{\rho R_o^2} t_2)\ddot{v} + (\frac{4\pi R_o c_1}{\rho} + \frac{4\mu}{\rho R_o^2} + t_1\omega^2)\dot{v} + \omega^2 v = 0 \quad (3.10)$$

Equation (3.10) is the equation of motion of small amplitude bubble pulsation in an Oldroyd liquid. Yang and Lawson (1974) derived a similar equation for bubble pulsation but they did not include the thermal and acoustic radiation terms. If $t_1 = t_2 = 0$ Equation (3.10) is reduced to its Newtonian form

$$\ddot{v} + (\frac{4\pi R_o c_1}{\rho} + \frac{4\mu}{\rho R_o^2})\dot{v} + \omega^2 v = 0 \quad (3.11)$$

A comparison of Equation (3.10) and (3.11) shows that the visco-elasticity of the liquid has changed a second order equation (3.11) into a third order one (3.11) It also shows that visco-elasticity has resulted in the addition of new terms.

The behaviour of the system described by Equation (3.10) would be better understood by solving the characteristic equation of the system. The characteristic equation of the system is

$$D^3 + B_1 D^2 + B_2 D + B_3 = 0 \quad (3.12)$$

where

D = differential operator

$$B_1 = (1 + \frac{4\pi R_o c_1 t_1}{\rho} + \frac{4\mu t_2}{\rho R_o^2}) / t_1$$

$$B_2 = (\frac{4\mu R_o c_1}{\rho} + \frac{4\mu}{\rho R_o^2} + t_1\omega^2)/t_1$$

$$B_3 = \omega^2/t_1$$

The roots of the characteristic Equation (3.12) are

$$D = -a_o, -a_1 + jb_1, -a_1 - jb_1 \quad (3.13)$$

where j is $(-1)^{\frac{1}{2}}$ and

$$a_o = \frac{1}{3} B_1 - (A + B)$$

$$a_1 = \frac{1}{3} B_1 + \frac{1}{2}(A + B)$$

$$b_1 = \frac{3^{\frac{1}{2}}}{2} (A - B)$$

$$A = (-\frac{1}{2} b + Y^{\frac{1}{2}})^{1/3}$$

$$B = (-\frac{1}{2} b - Y^{\frac{1}{2}})^{1/3}$$

$$Y = (\frac{1}{4}b + a^3)$$

$$a = \frac{1}{9} (3B_2 - B_1^2)$$

$$b = \frac{1}{27} (2B_1^3 - 9B_1B_2 + 27B_3) \qquad (3.14)$$

The nature of the roots depends on the value of the discriminent Y being greater than, equal to or less than zero. The case of interest to us is $Y > 0$. In this case, there will be one real root and two conjugate complex roots. The solution to Equation (3.10) would then be

$$v = A_1 \exp(-a_o t) + A_2 \exp(-a_1 t) \sin (b_1 t + \phi) \qquad (3.15)$$

where A_1, A_2 and ϕ are constants.

As seen from Equation (3.15) the pulsation of the bubble in an Oldroyd liquid consists of an exponentially decaying component and a damped sinusoidal component of natural frequency b_1. The amplitude of pulsation decays with a time constant $1/a_1$.

In contrast, the solution to Equation (3.11) is

$$v = A_3 \exp(-a_n t) \sin (\omega t) \qquad (3.16)$$

where A_3 is a constant and a_n is the Newtonian damping constant. The amplitude of pulsation decays with a time constant $\frac{1}{an}$. Our main interest is to find the effect of visco-elasticity on the damping constant a_n when changed to a_1 in visco-elastic liquids.

3.3 EFFECTS OF ELASTICITY ON THE NEWTONIAN DAMPING CONSTANT OF PULSATING BUBBLES

In Figures (3.1), (3.2) and (3.3) we present some results of the effects of the elasticity of visco-elastic liquids on the Newtonian damping constant of pulsating bubbles. The results are the solutions to Equations (3.15) and (3.16). The equations were solved on the computing facilities of the Edinburgh Regional Computing Centre (ERCC). The computer program (see Appendix A) was written in FORTRAN IV language.

In Figure (3.1) we present some results on the effects of the relaxation time of the liquid on the damping constant of a bubble 0.3 cm in radius (f_o = 1080 Hz). As shown in the figure, the elasticity of the liquid reduces the value of the Newtonian damping constant The effectiveness of the elasticity in reducing the damping of bubble pulsation depends on the relaxation time of the liquid, and increases the greater the relaxation time.

In Figure (3.2) we present some results on the effect of the relaxation time on the pulsation damping of bubbles of different sizes. The bubble radii are 3 and 0.003 cm (f_o = 108 Hz and 108 KHz respectively). The figure shows that the damping constant of the large bubble increases as the relaxation time becomes orders of magnitude smaller than the bubble time constant $1/f_o$. At a relaxation time of 10^{-5} sec, which is small in comparison with the time constant of the large bubble and of the same order of magnitude of the time constant of the small bubble, the damping constant of the large bubble has increased sharply while that of the small bubble has been very much reduced. It is also shown that at relaxation times higher than the bubble time constant, the damping constant asymptotes. The value to

which the damping constant asymptotes is that of a damping constant of a bubble pulsating in an inviscid liquid.

In Figure (3.3) we present some results on the effects of the retardation time of the liquid on the damping of bubble pulsation. The bubbles have radii of 0.3 and 0.003 (f_o = 1080 Hz and 108 KHz respectively). The figure shows that the greater the retardation time the greater the damping constant. An interesting result of the effects of the retardation time is that when it has a value equal to that of the relaxation time the damping constant recovers its Newtonian value. However, the retardation time of a visco-elastic liquid is in general smaller than the relaxation time which means that visco-elastic liquids would still reduce the damping constant of bubble pulsation.

To sum up, the relaxation time is an effective reducer of bubble pulsation damping when it is of a comparable value of the bubble time constant and behaves oppositely when it is an order of magnitude smaller. The effects of the retardation time oppose those of the relaxation time and when both have the same value the liquid behaves in a Newtonian way.

CHAPTER 4

AN ACCOUNT OF THE WORK ON LIQUID COLUMN OSCILLATION AND COATED TUBES

4.1 INTRODUCTION

Two topics will be investigated in this Chapter: The effects of the adsorbed layers of polymer on the skin friction of turbulent shear flow and liquid column oscillations, and the effects of the drag-reducing polymer solutions and surfactants on the damping of liquid column oscillations. Below, we will present a survey of the relevant works on the damping of liquid column oscillations followed by another survey on the effects of the adsorbed layers of polymers on the turbulent shear flows and the damping of liquid column oscillations. McComb's report (1974) on the effects of polymer additives and polymer adsorbed layers on the damping of liquid column oscillations is considered as the connecting link between the two topics.

The results of our investigations will be presented in reverse order, ie. we will start with the results of the effects of the adsorbed layers of polymers on turbulent shear flow followed by the results of the effects of drag-reducing polymer additives and surfactant additives on the damping of liquid column oscillations. After this we will present the effects of adsorbed layers of polymers on the damping of liquid column oscillations with and without polymer additives.

The reason for this arrangement is that this work started as an extension of McComb's work (1974) with the aim of assessing the effects of the adsorbed layers of polymer on turbulent shear flow. Following the results obtained, it was decided to expand the investigation on the damping of liquid column oscillations.

4.2 THE DAMPING OF LIQUID COLUMN OSCILLATIONS

The damping of liquid column oscillations is usually studied in U-shaped tubes. The liquid in the manometer (tube) is given an initial displacement and then released to oscillate freely. The amplitude of successive oscillations decreases with time due to viscous friction mainly.

A sketch of a U-tube manometer is shown in Figure (4.2). At rest, the liquid in both sides of the manometer is at the same level. The liquid column is given an initial displacement of Z_o, on one side of the manometer, above the equilibrium level. The task that faces any researcher is to find the equation of motion of the liquid column oscillation.

The equation of motion of a real incompressible fluid is cylindrical coordinates (Pai (1956)) is

$$\rho \frac{DU_z}{Dt} = F + \frac{1}{r}\left[\frac{\partial r\tau_{rz}}{\partial r} + \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial r\delta_{zz}}{\partial z}\right] \qquad (4.1)$$

where

F = external body force per unit volume

τ = shear stress

δ = normal stress

r = radial distance

Subscripts:

r, θ, z = directions of cylindrical coordinates

Also, in cylindrical coordinates

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U_r \frac{\partial}{\partial r} + \frac{U_\theta}{r} \frac{\partial}{\partial \theta} + U_z \frac{\partial}{\partial z} \qquad (4.2)$$

The liquid column is assumed to have an axial velocity component only. The axial velocity is a function of time and the radial distance from the centre of the tube. All derivatives with respect to θ vanish. Deviatoric normal stress tensors vanish as well. Equation (4.2) is thus reduced to

$$\frac{D}{Dt} = \frac{\partial}{\partial t} \tag{4.3}$$

In the absence of external forces, Equation (4.1) and (4.3) together give

$$\rho\frac{\partial U_z}{\partial t} = \frac{1}{r}\left[\frac{\partial r\tau_{rz}}{\partial r} + \frac{\partial r\delta_{zz}}{\partial z}\right] \tag{4.4}$$

Equivalently

$$\rho\frac{\partial U_z}{\partial t} = \frac{\partial \tau_{rz}}{\partial r} + \frac{\tau_{rz}}{r} + \frac{\partial \delta_{zz}}{\partial z} \tag{4.5}$$

For a Newtonian fluid

$$\tau_{rz} = \mu e_{rz} \tag{4.6}$$

$$\delta_{zz} = -p + \sigma_{zz} \tag{4.7}$$

$$\sigma_{zz} = 2\mu e_{zz} \tag{4.8}$$

Since the axial velocity is independent of the axial distance there results

$$e_{zz} = 0 \tag{4.9}$$

For a general fluid

$$e_{rz} = \frac{\partial U_z}{\partial r} \tag{4.10}$$

Subsituting Equations (4.6), (4.7), (4.8), (4.9) and (4.10) in Equation (4.5) we obtain

$$\rho \frac{\partial U_z}{\partial t} = \frac{\partial}{\partial r} \left(\mu \frac{\partial U_z}{\partial r} \right) + \frac{\mu}{r} \frac{\partial U_z}{\partial r} - \frac{\partial p}{\partial z} \tag{4.11}$$

hence

$$\rho \frac{\partial U_z}{\partial t} = - \frac{\partial p}{\partial z} + \mu \left(\frac{1}{r} \frac{\partial U_z}{\partial r} + \frac{\partial^2 U_z}{\partial r^2} \right) \tag{4.12}$$

For an inviscid liquid, the viscosity term vanishes and the axial velocity becomes a function of time only. Equation (4.12) becomes

$$\rho \frac{\partial U_z}{\partial t} = - \frac{\partial p}{\partial z} \tag{4.13}$$

where

$$U_z = \frac{dz}{dt} \tag{4.14}$$

Biery (1963) showed that

$$\frac{\partial p}{\partial z} = \frac{2 \rho g z}{L} \tag{4.15}$$

where

z = height of liquid column above equilibrium level

L = total length of liquid column.

Subsituting Equations (4.14) and (4.15) in Equation (4.13) there results

$$\frac{d^2 z}{dt^2} + \frac{2g}{L} z = 0 \tag{4.16}$$

Equation (4.15) describes a simple harmonic system. The resonance frequency of the system

$$\omega_o^2 = \frac{2g}{L} \tag{4.17}$$

For a real fluid, the complete solution of Equation (4.5) is

required. However, for a Newtonian fluid with a laminar flow regime and a constant coefficient of viscous friction, the equation of liquid column oscillations takes the form

$$\ddot{z} + 2\xi\omega_o \dot{z} + \omega_o^2 z = 0 \qquad (4.18)$$

where

ξ = damping constant of liquid column oscillation

In a turbulent flow regime Equation (4.18) no longer holds. The equation of liquid column oscillations takes the form

$$\ddot{z} + c\,\dot{z}^n + \omega_o^2 z = 0 \qquad (4.19)$$

where

c = damping coefficient

n = turbulent damping exponent

Different investigators have tackled the problem of the damping of liquid column oscillations and derived different expressions for the damping constant. The investigators were also concerned with defining a critical Reynolds number above which the flow would become turbulent rather than laminar. Below we deal with laminar damping only.

Bird, Stewart and Lightfoot (1960) tackled the problem by assuming a time varying parabolic velocity profile. On the basis of mechanical energy balance they derived an equation for the damping of liquid column oscillation identical to Equation (4.18). Their damping constant has the form (Biery (1969))

$$\xi = \sqrt{12}\,(N_r)^{-1} \qquad (4.20)$$

where

$$N_r = \frac{\omega_o R^2}{\nu} = \text{Valensi number}$$

R = radius of the manometer tube

ν = kinematic viscosity

Experimental results (Biery (1969)) showed that the damping constant as defined by Equation (4.20) gave a good asymptote near the critical damping region. In other regions, it predicts low values of the damping constant.

Valensi (1947) pointed out that the flow regime of an oscillating liquid column depends on the dimensionless parameter $\frac{\omega_o R^2}{\nu}$ (now known as the Valensi number). For a Valensi number less than 20, the flow is laminar with a parabolic velocity profile. At a Valensi number larger than 20 and less than 70, the velocity profile is non-parabolic. For higher values there would exist a well-defined central core and a developed boundary layer.

The first theoretical calculations of the damping constant were made by Valensi (1947) who derived the following expression (Biery (1969))

$$\xi = 2.892\ (N_r)^{-1} \tag{4.21}$$

The expression of Equation (4.21) was modified later (Valensi (1947)) to

$$\xi = 3.9536\ (N_r)^{-1} \tag{4.22}$$

Valensi and Von Karman (1948) applied boundary layer theory to the liquid column oscillation and obtained an asymptotic solution for small damping factors. They derived the following expression for the damping constant

$$\xi = \frac{1}{\sqrt{2}}\ (N_r)^{-\frac{1}{2}} \tag{4.23}$$

The experimental results of Menneret (1911) and Biery (1963) showed that the boundary layer solution gave a good asymptote for high

Valensi number ($N_r > 500$).

In the light of this information, later investigators (Ury (1962) and Biery (1963) resorted to solving Equation (4.12) numerically and presented their results in forms of graphs. Ury (1962) obtained a solution for the equation of a damped oscillatory system (Equation (4.18)) coupled with the equation of motion. His results in the form of a graph are re-plotted in Figure (4.8). Biery (1963, 1969) presented a numerical solutions of Equation (4.18). His calculated results were obtained by numerically integrating the axial component of the equation of motion and are re-plotted in Figure (4.8). Figure (4.2) shows clearly that Ury's damping constant is higher than Biery's.

All the experimental results on the damping of liquid column oscillation were obtained by measuring the damping of a liquid column in a U-tube. In such a configuration of the liquid column the damping of oscillation will be affected by the curvature of the tube, the reversal of flow direction and surface tension. All the above mathematical and numerical solutions neglected these effects. Consequently, the reported experimental results of the damping constant showed a deviation from the calculated values.

Biery (1969) presented experimental data which covered a wide range of Valensi numbers. His results showed that the laminar velocity profile solution and the boundary layer solution gave good asymptotes for the damping constant at low and high Valensi numbers respectively. In the intermediate region, the experimental results were slightly higher than his calculated ones. Biery attributed that to the end effects (reversal of flow direction) which, he found, would adjust the damping constant from 5-15%. The larger the tube diameter

the higher the end effects. Biery's experiments were carried out in a glass manometer.

Ury (1962) presented the experimental data for the damping of a mercury liquid column in a plastic tube. The experimental results agreed with his calculations but were significantly higher than Biery's. Park and Baird (1970) investigated the damping of liquid columns of mercury, water, methanol and toluene in a glass manometer. The damping of the mercury column agreed with Ury's calculations while the rest of the fluids agreed with Biery's. Park and Baird observed that all the liquids they used wetted the surface of the glass tube except the mercury. Chan and Baird (1974) measured the rates of energy dissipation in an oscillating liquid column. They found that the measured rates were up to 150% greater than the predictions of the laminar boundary layer theory. They attributed the discrepancy between the experimental and theoretical results to periodic turbulence.

4.3 THE EFFECTS OF SURFACTANTS AND POLYMER ADDITIVES ON THE DAMPING OF LIQUID COLUMN OSCILLATION

The effects of drag-reducing polymer additives on the damping of liquid column oscillation have not yet been investigated extensively. Biery (1964) investigated the damping of liquid column oscillation of polyox, WSR-301, solution of high concentration (·8 and 1% respectively). Biery did not find any anomalous disagreement between his experimental and calculated results that may be attributed to the visco-elastic characteristics of the fluid. He concluded that steady state rheological data of the non-Newtonian liquids adequately described the behaviour of the liquids in low frequency unsteady oscillatory flow. Biery's conclusion was based on the fact that the cycle time of the

liquid column oscillation ranged between 1.5 - 2 sec while the retardation and relaxation times of the polymer were 5×10^{-3} and 22×10^{-3} respectively. Consequently, the elasticity of the solution will exhibit no noticeable effects. Biery's experiments were carried out in a glass tube manometer.

McComb (1974) studied the effects of drag-reducing polymer additives on the damping of liquid column oscillations in a (1.3 cm nominal bore) flexible clear PVC tube. The polymers used were Polyethylene oxide (WSR-301 and N-3000) and Polyacrylamide (Separan AP 273 and MG 200). The concentration of the solutions ranged from 0-2000 wppm. The effects of the polymer additives on the damping of the liquid column oscillation were assessed by observing the percentage increase in the total number of oscillations relative to the water. The percentage increase in the number of oscillations was defined as

$$100 \frac{(Np - Ns)}{Ns} \tag{4.24}$$

where

Ns = number of oscillations of solvent column

Np = number of oscillations of polymer column

McComb's experiments were carried out in two manometers. Both manometers were made of flexible PVC tubes of 1.3 cm nominal bore. One of the manometers had a radius of curvature of 29 cm whereas the other had a 4.5 cm radius of curvature. The length of liquid column in both configurations was 88 cm.

McComb's results showed that low concentrations of Polyox WSR-301 additives increased the total number of oscillations of the liquid column significantly. In the large radius of curvature configuration,

the maximum percentage increase was 120% while in the small radius of curvature it was 30-40%. Polyox N-3000 increased the number of oscillations but less significantly. Polyacrylamide additives exhibited no noticeable effects.

McComb (1974) reported that polyox additives were adsorbed on the surface of the PVC tube. The manometers with an adsorbed layer (coated tube) of polyox increased the total number of oscillations of a water column relative to a clean tube. Polyacrylamide additives did not coat the PVC tube.

Ayyash and McComb (1976) investigated the effects of Aerosol solutions on the damping of liquid column oscillation in a PVC tube. They also investigated the effects of Polyox WSR-301 and Aerosol additives on the damping of liquid column in a glass tube. Their results showed that the Aerosol additives increased the total number of oscillations of a water column in a PVC tube. The effects of the Aerosol additives were quantitatively much less than the WSR-301 additives. In a glass tube both additives showed no effect on the number of oscillations at low concentrations. At high concentrations there was a reduction in the number of oscillations.

Another matter of interest in the field of liquid column oscillation is the transition phenomenon of the flow field of an oscillating liquid column. This was investigated by Ury (1962), Sergeev (1966) and Park and Baird (1970). The transition criterion was taken to be the time maximum Reynolds number defined as

$$Re_{max} = \frac{2Z_o\omega_o R}{\nu} \qquad (4.25)$$

where

Z_o = maximum displacement of liquid column

According to Sergeev (1966)

$$Re_{crit} = 990\ \beta \tag{4.26}$$

and according to Park and Baird (1970)

$$Re_{crit} = 450\ \beta^{4/3} \tag{4.27}$$

where

$$\beta = R\ (\frac{\omega}{2\nu})^{\frac{1}{2}}$$

As will be shown later, our experiments were carried out at Reynolds numbers below the critical value.

4.4 TURBULENT DRAG-REDUCTION BY ADSORBED LAYERS OF POLYMERS

Different hypotheses have been postulated by different investigators to explain the phenomenon of drag-reduction by long chain polymer additives (see the discussion in Chapter 1). Among these hypotheses, an adsorbed layer of polymer on the wall of the tube was proposed as a potential mechanism of drag-reduction. However, this hypothesis is still a subject of controversy among researchers in drag-reduction. Below, we will present some of the views expressed in this field.

The early observations of the effects of adsorbed layers of polymer on the skin friction of turbulent shear flow were carried out by observing the time required to "wash off" the effects of such layers, in tubes previously tested with drag-reducing solutions. El'Perin and Smolskii (1965) reported a persistent decrease in the friction factor up to the fourth change of water. Davis and Ponter (1966) reported the persistence of drag-reduction for 15 minutes after the polymer solution was replaced by a base solvent.

Little (1969) investigated the effects of adsorbed layers of polymers and pointed out that the persistence of drag-reduction in the presence of the solvent was attributed to the "bleeding" of entrapped polymer solutions in the pressure tap connections. However, Little concluded that depending on the flow rate and initial polymer concentration, the residual polymer adhering to the pipe wall appeared to be swept away in a time period only three to four times greater than that required to displace the bulk of the polymer solution.

Arunachalam and Fulford (1971) presented experimental data indicating increased additive concentrations at the pipe wall and concluded that adsorption of polymers on the pipe wall occurs under turbulent flow condition. The adsorbed layers, in their view, were responsible for the drag-reduction. Hand and Williams (1973) demonstrated the existence of adsorbed-entangled layers of polymer at the flow boundaries of dilute polymer solutions. They found that layers of high molecular weight polymers are harder to remove, ionic polymer layers are easier to wash off and layers formed by dilute polymer solutions are easier to remove. They also reported "anchoring" entangled layers of polymers which exhibited semi-permanent drag-reduction in the presence of solvents.

Gyr and Mueller (1974) investigated the effects of adsorbed layers by carrying out tests in two identical tubes made from strong and weak adsorbing materials respectively. Their results showed that adsorbed layers of polymer played a secondary role - if any - in drag-reduction. They suggested that Hand and Williams' (1973) results were an example of the compliant-wall (Dolphin skin effect) rather than a turbulent drag-reduction mechanism.

Ayyash and McComb (1976) investigated the effects of adsorbed

layers of Polyox WSR-301 on turbulent shear flow in a flexible PVC tube. McComb (1974) reported that such adsorbed layers increased the number of oscillations of a water column relative to a clean non-coated tube. The results of Ayyash and McComb (1976) showed inconsistent effects of the adsorbed layer; drag increase as well as drag decrease were reported.

4.5 EXPERIMENTAL SET-UP

The experiments relevant to the work discussed in this chapter were carried out on two different experimental set-ups. The first set-up was to investigate the effects of adsorbed layers of polymer on turbulent shear flow. The other was to investigate the effects of additives and adsorbed polymer layers on the damping of liquid column oscillations. A brief description of both set-ups is presented below.

A schematic diagram of the first set-up is shown in Figure (4.1). The water was circulated in a closed loop using a centrifugal pump. A flow meter was installed to measure the flow rate. Three tubes flanged together represented the whole test section. The upstream section was used to check the pressure drop of water in a clean tube. The centre section was the coated tube where the effect of adsorbed polymer layers were expected to exhibit some effects. The downstream section was intended to monitor the expected "wash off" of the adsorbed layer from the centre section. Each of the three sections had two pressure taps. The distances between the pressure taps on each section were equal. Tube materials were restricted to glass or transparent plastics because it was intended to use laser doppler anemometry to study the flow structure.

All coated tubes (with one exception described later) were

prepared by filling them with 2000 wppm aqueous solutions of Polyox WSR-301 and leaving them for a week. The tubes were then emptied and left to dry for a further week. The experiments were carried out by measuring the pressure drop across the three test sections. Discussion of the results will follow in a later section of this chapter.

The set-up to investigate the damping of liquid column oscillation is shown in Figure (4.2). A U-shaped tube was fixed to a wooden board. Two tube materials were used, flexible PVC tubes 1.25 cm ID and a glass tube 1.1 cm ID. The radius of curvature of the glass manometer was 29 cm. The PVC tube was used in two different configurations of 29 and 4.5 cm radii of curvature respectively. The length of the liquid column in all cases was 141 cm.

All experiments were carried out by giving the liquid column an initial displacement of 25 cm and then releasing it to oscillate freely. The peaks of the successive oscillations were measured from graduations on the surface of the tube - several runs were taken for every experiment. The values of the peaks were measured and averaged. The results of a typical test are listed in Table 4.1. The average cycle time was measured by measuring the total time of several successive oscillations and then finding the average value of the cycle time. The average cycle time was 1.8 sec compared with a theoretical value of 1.7 sec.

The test solutions were water, Polyox WSR-301 solutions, Polyacrylamides AP 273 solutions and Aerosol solutions. Coated PVC tubes were also tested using water and Polyox WSR-301 solutions. The results of the tests are discussed below.

4.6 DISCUSSION OF RESULTS

The experimental results are presented and discussed below in the following order:

1. The effect of adsorbed layers of polymers on turbulent shear flow.
2. The effect of polymer and surfactant additives on the damping of liquid column oscillation in a flexible PVC tube.
3. The effect of adsorbed layers of polymer on the damping of liquid column oscillation in a flexible PVC tube.
4. The effect of polymer and surfactant additives on the damping of liquid column oscillation in a glass tube.

4.6.1 The Effect of Adsorbed Layers of Polymer on Turbulent Shear Flow

The experimental results of the effects of adsorbed layers of Polyox WSR-301 are shown in Figures (4.3) and (4.4). Figure (4.3) shows the effect of a well-aged adsorbed layer of Polyox on turbulent shear flow. The adsorbed layer reduced the friction factor of turbulent shear flow in a flexible PVC tube. The friction factor was reduced by about 30%. Figure (4.4) shows the effect of a freshly prepared coated tube (fresh adsorbed layer) on shear flow. It is clear, in this case, that the adsorbed layer had no effect on the friction factor. However, the coated tube which gave no effect on turbulent shear flow was subsequently tested in a manometer configuration to assess the effect of the coating layer on the damping of a water column. The results of the test are shown in Figure (4.5). The results indicate that the adsorbed layer retained its effectiveness in reducing the damping of water column oscillation (McComb (1974)).

The adsorbed layer of Polyox did not seem to have been washed off in shear flow.

A possible explanation of the discrepancy in the results of the effect of adsorbed layers is that an adsorbed mono-layer of polymer may be sufficient to reduce the damping of liquid column oscillation whereas in turbulent shear flow an adsorbed-entangled layer is necessary to reduce the turbulent drag. The coated tube which reduced turbulent drag was prepared in the course of a test of concentration in a manometer oscillation experiment (McComb (1974)). The adsorbed layer in this case was the result of a series of solutions of increasing concentration. The adsorbed layer which showed no effect was prepared by filling the tube with a Polyox WSR-301 solution of 2000 wppm concentration.

Hand and Williams, in preparing their coated tubes, passed successively poorer solvents in the tubes in order to obtain permanent drag-reducing layers. Their process might be thought to have some resemblance to the process of successively increasing the concentration of the test solution in a PVC manometer (McComb (1974)).

In conclusion, we may say that layers of Polyox WSR-301 are adsorbed on the surface of flexible PVC tubes. Such layers seem to be of a semi-permanent nature and are not washed off by turbulent shear flow. This makes them a favourable subject for further investigations on the effect of adsorbed layers on turbulent drag. The adsorbed layers of Polyox WSR-301 showed consistent effect in reducing the damping of liquid column oscillation whereas in turbulent shear flow they yielded inconsistent results.

4.6.2 The Effect of Polymer and Surfactant Additives on the Damping of Liquid Column Oscillations in a Flexible PVC Tube

In investigating the effects of polymer additives on the damping of liquid column oscillation, McComb (1974) measured the effectiveness of polymer additives by calculating the percentage change in the total number of oscillations of a polymer solution relative to a pure solvent. According to McComb (1974) the effectiveness of the polymer additive is expressed as

$$100\ \frac{(Np - Ns)}{Ns}$$

where

Np = number of oscillations of a polymer solution column

Ns = number of oscillations of a solvent column.

Previous investigators (Ury (1962), Biery (1963) and Park and Baird (1970)) measured the damping constant by solving Equation (4.18). The solution of Equation (4.18) in terms of ξ, the damping constant, is (Biery (1963))

$$\frac{\xi^2}{1 - \xi^2} = \left[\frac{1}{n\pi} \ln \left| \frac{Z_o}{Z_n} \right| \right]^2 \qquad (4.28)$$

For $\xi << 1$, Equation (4.28) becomes

$$\xi = \frac{1}{n\pi} \ln \left| \frac{Z_o}{Z_n} \right| \qquad (4.29)$$

where

Z_o = reference amplitude

Z_n = oscillation amplitude after n half cycles

n = number of half cycles between Z_o and Z_n.

The damping constant of liquid column oscillation in

non-turbulent flow regimes was presented, in the form of graphs, as a function of Valensi numbers (Biery (1969)). In Figure (4.8) the results of Ury (1962) and Biery (1963) are re-plotted for the range of Valensi numbers of interest. In this section, we present our results according to both methods of analysis, McComb's method and the method of Biery et al.

McComb (1974) reported that low concentration of Polyox WSR-301, when added to water, more than doubled the total number of sscillations of a water column in a PVC tube under the same conditions. He also reported that Polyox additives were more effective in manometers of larger radii of curvature. Polyacrylamide additives were reported to have exhibited no effects on the total number of oscillations.

We carried out tests to assess the effect of Polyox and Aerosol additives on the oscillation of a liquid column. We observed that Polyox WSR-301 additives increased the total number of oscillations of a water column in a PVC tube as reported by McComb (1974). The results of the Polyox WSR-301 test are not repeated here. In Figure (4.6) we present the effects of Aerosol additives on the total number of oscillations of a water column in a PVC tube. The tube was mounted in two different configurations, 29 cm and 4.5 cm radii of curvature respectively. In the same graph, the effects of Polyox N-3000 (a weak drag-reducer in turbulent shear flow) are presented for comparison purposes. It is clear from Figure (4.6) that Aerosol additives increased the total number of oscillations of a water column in a PVC tube. However, the effects of Aerosol additives are much less than the effects of Polyox WSR-301 additives (McComb (1974)) and

singificantly less than the effects of Polyox N-3000 additives. The graph also shows that the radius of curvature was virtually irrelevant in the case of the Aerosol additives while in the case of the Polyox additives it was important. McComb (1974) reported that for a radius of curvature of 4.5 cm the Polyox WSR-301 increased the number of oscillations by 30-40% whereas for a radius of curvature of 29 cm the increase was 120%. A possible explanation of this observation is that in small radii of curvature the possibility of radial velocity components in the curved section becomes bigger. Polyox solutions are reported to exhibit strong resistance to stretching (Oliver (1973)). Consequently, the possibility of large viscous losses in small radii of curvature becomes bigger. The firm conclusion to emerge is that Polyox WSR-301 additives are more effective than Aerosol solutions in increasing the total number of oscillations of water column in a flexible PVC tube. It required few parts per million by weight of Polyox WSR-301 to more than double the totalnnumber of oscillations whereas it required much higher concentrations of Aerosol additives to exhibit a much smaller effect.

However, when the experimental data are presented in the form of the effect of polymer and Aerosol additives on the damping constant of liquid column oscillation, a different picture emerges. If the amplitudes of successive oscillations of a laminar damped system (Equation (4.18)) are plotted on a log-linear graph paper they would fall on a straight line whose slope is a direct indication of the damping constant. In Figure (4.7) we present a plot of the amplitudes of successive

oscillation versus the number of cycles. It is shown in the graph that non-linear damping occurs in the first half cycle and at cycles of very low amplitudes. This behaviour of the liquid column was observed by other investigators (Ury (1962), Biery (1963) and Park and Baird (1970)). In the first half cycle, non-linearity occurs because of the secondary effects of starting up, whereas at low amplitudes secondary effects (flow direction reversal and surface tension) become large in comparison with the inertial effects. In calculating the damping constant we used Equation (4.29). The first half cycle was neglected (Biery (1963)).

Previous investigators observed changes in the damping constant per half cycle, Our calculations indicated the same trend. In Table 4.2 the damping constants per half cycle are presented. The table shows a fluctuation of the per half cycle damping constant around a mean value. The average damping constant calculated in this work was the average damping constant of 10 consecutive half cycles after the first half cycle. In polymer solutions of relatively high concentrations, it was not possible to obtain data for 10 half cycles. Consequently, we used 6 consecutive half cycles after the first half cycle. In Table 4.2 we present the average damping constant based on 10 half cycles and 6 half cycles respectively. As it is shown, the error involved in taking less numbers of half cycles is within the overall experimental error.

In Figure (4.8), we present the results of the calculations of Ury (1962) and Biery (1963) as reported by Biery (1969) in the range of Valensi numbers of interest in this work. In the

same graph we present the result of the damping constant of the oscillation of a water column in a flexible PVC tube. As shown, our results are more in line with Ury's calculations than with Biery's. Ury (1962) presented his results of the damping of a mercury column in a plastic tube. His results were in line with his calculations. Biery (1969), whose calculations, neglecting the end effects and surface tension effects, gave lower damping constants speculated that large secondary effects probably have increased the damping constant of Ury's experimental results. He added that if the plastic tube was flexible then possibly added dissipation could occur in the movement of the tube itself. Park and Baird (1970) observed that the damping constant of a mercury column in a glass tube was in line with Ury's calculations whereas other liquids (water, methanol and tolouene) results were in line with Biery's. Park and Baird observed that all liquids except mercury wetted the surface of the tube. It seems likely, then, that the damping of a non-wetting agent is higher than a wetting agent, and that the failure of the mercury to wet Ury's tube is partially responsible for the high damping constant he obtained.

In our experimental work we observed that the wetting of the flexible PVC tube was very poor when water was used. Consequently, a higher damping constant would be expected. Other factors that would contribute to the increased damping is the possible movement of the tube wall because of its flexibility. The necking of the flexible tube when mounted in a U-shaped probably contributes towards increasing the damping constant.

In Figures (4.9) and (4.10), we present the results of the

effects of Polyox WSR-301 and Aerosol additives on the damping constant of water column oscillation. The results of Ury and Biery in the graphs are those of liquids of equivalent viscosity of Polyox and Aerosol solutions. The viscosities of the Polyox solutions are those of Table 5.2. The viscosities of the Aerosol solutions are shown in Figure (4.11) and were measured experimentally using a Ubbelohde viscometer at 20°C.

In Figure (4.9), the effects of Polyox WSR-301 on the damping constant of a water column in a flexible PVC tube are presented. It is shown in the graph that Polyox WSR-301 solutions of concentrations of 10 - 500 wppm have decreased the average damping constant by about 15 - 25% in comparison with Ury's results, the basis of our calculation in this section. However, at low concentrations the damping constants were less than Ury's calculations but higher than Biery's. At higher concentrations the damping constants approached the calculations of Biery.

Depending on the experimental results of Ury (1962), Biery (1963), Park and Baird (1970) and our results of Figure (4.8), it is possible to suggest that Biery's calculations (1969) represent the damping constant of a liquid column neglecting the end effects and surface tension effects as reported by Biery, whereas Ury's calculations represent the damping constant of a liquid column including the end effects and the surface tension effects. Biery (1963) calculated the effects of the end effects on the damping constant and found that these effects would adjust the damping constant by 10% on average. It would seem likely then that the surface tension effects adjust the damping constant by an equal amount approximately. According to Biery, the

retarding force because of the surface tension may be expressed as

$$F_{ST} = 2\pi R_o K_{ST} (\cos \phi_R - \phi_A) \qquad (4.30)$$

where

K_{ST} = surface tension

R_o = radius of the tube

ϕ_R = contact angle of receding column

ϕ_A = contact angle of advancing column.

If the tube is wetted by the liquid column both the contact angles approach zero and consequently, the surface tension effects become negligible. For a non-wetting tube, surface tension exhibits its effects due to the difference between the advancing and receding contact angles.

We did not measure the contact angles of advancing and receding columns but instead observed the wetting of the tube surface. If a liquid wets the tube then a thin film of liquid will be left behind the receding column. In the flexible tVC tube, the wetting was very poor and few drops of water formed behind the receding column. When the polymer solutions were added, the wetting very much improved and a liquid film formed behind the receding column. The improving of the wetting of the PVC tube was observed with the Aerosol additives. Ayyash and McComb (1976) reported similar observations. No measurements of the surface tension were made in this work. However, Oliver (1966) reported surface tension measurements of Polyox WSR-301 and Separan AP273 at different concentrations (.01, .05, .5%). Oliver's results showed similar reduction of the surface tension due to the

addition of polymers in water. The surface tension of water was reduced from 72 dyne/cm to 45-50 dyne/cm for both additives. His estimated values of the surface tension were about 60 dyne/cm for the above solutions. McComb and Ayyash (1976), in studying the behaviour of bubbles in water and polymer solutions, observed a reduction in the size of the bubble, formed at the tip of a brass nozzle situated horizontally, due to the addition of Polyox WSR-301 additives. When Separan AP 273 was used no reduction in the size of the bubble was observed. In contrast a slight increase in the size of the bubble was observed. According to Datta et al (1950), the size of the bubble is proportional to the cube root of the surface tension. Consequently, one would deduce from Datta's analysis and McComb and Ayyash's observations (1976) that the surface tension was reduced due to the addition of Polyox WSR-301 and that no reduction occured due to Separan AP 273 additives. McComb's results (1974) that Separan additives showed no effect on liquid column oscillation (at low concentrations at least) would indicate that the retarding force due to the surface tension did not change by the addition of Separan. Consequently, one would expect that Separan additives did not affect the surface tension, the contact angles or the end effects or that the net effects were negligible, and that Polyox additives did affect one or another of these factors. However, the results of McComb (1974) and our results (see below) on the effect of adsorbed layers of Polyox on liquid column oscillations would indicate that the Polyox additives did reduce the amount of retarding force of the surface tension. This reduction is probably attributed to both a reduction in the surface tension and an improved wetting.

Consequently, the reduction in the damping factor at low concentrations is probably due to a reduction in the surface tension, an improved wetting and possibly a suppression of fluid rotation at the end of the column. The further reduction in the damping factor at high concentrations would indicate that the end effect virtually disappeared. Biery (1963) and Park and Baird (1970) showed that the curvature of the tube had negligible effect on the damping of the liquid column. The radius of curvature in our configuration was higher than Biery's and Park and Baird's which makes it unlikely that the Polyox additives had any effect on the flow in the curved section.

The results of the effects of the Aerosol additives on the damping constant show that at low concentrations the Aerosol had little effect and that it required much higher concentrations of Aerosol than Polyox WSR-301 to reduce the damping constant by the same amount. This concentration dependence effect is analogous to the reported results of Ayyash and McComb (1976) and McComb (1974) on the effects of Polyox WSR-301 and Aerosol additives on the total number of oscillations of liquid column. It requires higher concentration of Aerosol to yield equivalent effects of much lower concentrations of Polyox, whether on the damping constant or the total number of oscillations of a water column in a PVC tube.

However, the results of McComb (1974), Ayyash and McComb (1976) and the results in this section show that although Polyox WSR-301 and Aerosol additives reduce the damping constant by equivalent magnitudes, the Polyox additives are much more efficient in increasing the total number of oscillations of the liquid column in a PVC tube.

4.6.3 The Effect of Adsorbed Layers of Polymer on the Damping Constant of Liquid Column Oscillation in a Flexible PVC Tube

McComb (1974) reported that Polyox WSR-301 was adsorbed on the surface of flexible PVC tubes. The total number of oscillations of a water column in a coated tube increased in comparison with the total number of oscillations in a clean (non-coated) tube (McComb (1974)); other parameters being the same. Our observations confirmed McComb's observations with regard to both the adsorption of Polyox WSR-301 and the increase in the total number of oscillations of a water column. In this section, we report the results of the effects of an adsorbed layer of Polyox WSR-301 on the damping constant of liquid columns of water and Polyox WSR-301 solutions.

By testing the Polyox solutions in coated PVC tubes, we aim at finding out the effects of the polymer additives, per se, on the damping of liquid column oscillation, ie. we aim at separating the effects of the bound polymers in the coated layers from the effects of the free polymers in solution. One may raise the question about the possibility of further adsorption of polymers by the coated PVC tubes and the possible relevance of increased adsorption on the damping of liquid column oscillation. In order to minimise the effects of further adsorption our tests were carried out in tubes which were tested previously in a series of increasing concentration of Polyox WSR-301 solutions. The solutions tested in the coated tubes were of equal concentrations to those tested in a clean tube. Further concentration-dependent adsorption is thus minimised.

The results of the effects of adsorbed layers of Polyox

WSR-301 on the damping of water and Polyox solutions column oscillation are presented in Figure (4.12). The experimental result of the damping constant of a water column in a coated PVC tube decreased in comparison with the experimental value of the damping constant in a clean tube. The result of a coated tube is more in agreement with Biery's calculations rather than Ury's. Since the adsorbed layer was reported to be insoluble in water (McComb (1974)) and not washable in turbulent shear flow (Ayyash and McComb (1976)), it seems likely that the reduction of the damping constant is primarily attributed to improved wetting of the tube surface. The further reduction in the damping constant with the increase of the Polyox concentration may reflect the contribution of these solutions in reducing the damping constant beyond improving the wetting of the surface.

The Polyox solutions in a coated tube would probably affect the end effects, generation of turbulence and possibly the structure of the flow. Further effects on the surface tension retarding force seems unlikely because the results of the water column in a coated tube show that the deviation of experimental results can be explained in terms of end effects. Consequently, the further reduction in the damping constant reflects the effect of Polyox additives on the secondary flows. As shown in Figure (4.12) these effects are quite small and the range of standard deviations of the results of Polyox solutions in coated and clear tubes overlap. The graph shows further that the Polyox solutions show no anomalous effects which may be attributed to the visco-elasticity of the polymer solutions (Biery (1964), McComb (1974)).

By comparing the results of Figures (4.9) and (4.12) it would

seem that the adsorbed layers of Polyox WSR-301 were slightly more effective in reducing the damping constant than dilute Polyox WSR-301 solutions. We showed above that these dilute solutions have a small effect on the other parameters affecting the flow other than the surface tension retarding force. It seems possible then, to conclude that the prime effects of the dilute polymer solutions are to improve the wetting of the surface of the tube and to reduce the surface tension. At higher concentrations the Polyox solutions seem to suppress secondary flow retarding forces. The results suggest that the adsorbed layers of Polyox WSR-301 probably improve the wetting of a flexible PVC tube better than dilute Polyox WSR-301 solutions.

4.6.4 The Effects of Polymer and Surfactant Additives on the Damping Constant of Liquid Column Oscillation in a Glass Tube

In order to clarify the results of the above tests and to give a more concrete explanation of the effects of Polyox and Aerosol additives on the damping of liquid column oscillation, tests were carried out in a glass tube manometer. Previous investigators (Biery (1963) and Park and Baird (1970)) investigated the damping of liquid column oscillation in glass tubes and reported consistent results. They both reported that water wetted the surface of the glass tube. B ry (1963) reported that the contact angles of the receding and advancing liquid columns in a glass tube were virtually zero degrees, and therefore, he neglected the effects of the surface tension retarding force in discussing his experimental results. We also observed that water wetted the glass tube.

In the above sections, we postulated that the decrease in

the damping constant of liquid column oscillation in a flexible PVC tube due to the addition of Polyox and Aerosol solutions is associated primarily with improving the wetting of the tube surface. In a glass tube, further improvement of the wetting of the surface would result in no effects on the damping of the liquid column since no changes in the contact angles are possible. Any reduction in the surface tension due to the additives would exhibit no effects on the damping since the surface tension retarding force of the water column is virtually zero anyway. Therefore, any reduction in the damping constant of the liquid column would be attributed mainly to the effects of the additives on the secondary flows and/or the structure of the bulk flow.

Having this in mind, we carried out tests using water and solutions of Polyox WSR-301, Separan Ap-273 and Aerosol. Separan AP-273 solutions were reported by McComb (1974) to have exhibited no effects on the total number of oscillations of a liquid column in a flexible PVC tube. McComb's results would indicate that Separan is a poor wetting agent. Aerosol solutions are good wetting agents but show no effects in turbulent shear flow (Ayyash and McComb (1976)). Polyox WSR-301 solutions are good wetting agents (see above sections) and good drag-reducers (Virk (1975)). Thus the solutions used in our test cover good combinations of liquid properties relevant to this work.

The results of the different tests are shown in Figure (4.13) and (4.14) and (4.15). In Figure (4.13) the results of the effects of the Aerosol additives on the damping of liquid column in a glass manometer are presented. Also shown in the same graph is the experimental result of the damping of a water liquid column

in a glass tube. The result of the water test showed that the damping constant is in agreement with the calculations of Biery rather than Ury. Our results confirm previous results of Biery (1963) and Park and Baird (1970). The plotted results of Biery and Ury are those of liquids of equivalent Valensi number to those of the Aerosol and polymer solutions as reported by Biery (1969). At low concentrations, Aerosol solutions showed no significant effects on the damping constant of the liquid column in a glass tube. At high concentrations the experimental results approached the calculated results of Biery, reflecting probably a suppression of the secondary flow effects. No change in the total number of oscillations of the liquid column were observed at low concentrations of Aerosol. At high concentrations, a reduction in the number of oscillations was observed reflecting the effects of increased viscosity.

In Figure (4.14), the results of the effects of Polyox WSR-301 additives are shown. The results indicated the same trend of the Aerosol additives qualitatively. The results of the Polyox WSR-301 would indicate that the visco-elasticity of the solutions had no noticeable effect on the damping of the liquid column oscillation. These results confirm Biery's results (1964) on the behaviour of concentrated Polyox WSR-301 solutions in a laminar damped oscillatory flow. The results seem to support our view mentioned above that the decrease in the damping of liquid column oscillation in a flexible PVC tube is associated with improved wetting of the tube surface rather than induced changes in the structure of the bulk flow. No changes in the number of oscillations of the liquid column were observed at low

concentrations but at high concentrations a reduction in the number of oscillations was observed.

Figure (4.15) shows the experimental results of the effects of Separan Ap-273 additives on the damping of liquid column oscillation in a glass tube. The results showed that the Separan additives had no noticeable effects on the damping. McComb (1974) reported that Separan additives had no effect on the total number of oscillations of a liquid column in a PVC tube. McComb's results would indicate that the wetting of the tube surface was not affected by the Separan additives. We did not observe any changes in the total number of oscillations due to the Separan additives in a glass tube. It would seem from our results, and McComb's results that Separan additives do not change the wetting of the tube surface either way. The decrease in the damping constant at high concentrations of Separan probably reflects the suppression of the effects of secondary flow.

Finally, we refer to the problem of transition from laminar to turbulent flow. The maximum Reynolds number obtained in the experiments was just below the predicted critical Reynolds numbers according to Equations (4.26) and (4.27). The maximum amplitude of the first half cycle was 17.5 cm. The time-maximum Reynolds number according to Equation (4.25) in the PVC tube is

$$Re_{crit} = 7620$$

According to Equation (4.26)

$$Re_{crit} = 8300$$

and according to Equation (4.27)

$$Re_{crit} = 7700$$

The results of the calculations of the critical Reynolds number show that the flow regime in our experiments did not exceed the critical Reynolds number according to both methods of calculation. The quoted results of the time-maximum Reynolds number is the maximum value obtained in our experimental work. Other values will be smaller due to the decrease in the amplitude of oscillation, the increase in the viscosity of the reduction in the size of the tube. Thus, it is satisfactory to assume that the results of our tests describe the damping of liquid column oscillation in laminar flow.

4.7 CONCLUSION

In the preceding sections of this chapter, we presented an account of the work we carried out to assess the effects of the adsorbed layers of Polyox WSR-301 on turbulent shear flow and on the damping of liquid column oscillation. An account of the work done to assess the effects of drag-reducing polymers and Aerosol additives on the damping of liquid column oscillation was also presented.

Our experimental results on the effects of adsorbed layers of Polyox WSR-301 on turbulent shear flow were inconsistent. Reduction in the turbulent drag was observed and the absence of any reduction in the turbulent drag in coated tubes was also observed. The inconsistency of our results was not altogether surprising in view of the reported results of other researchers, who reported conflicting results on the effects of the adsorbed layer of polymers on turbulent shear flow (Hand and Williams (1973), Gyr and Mueller (1974)). The available information on the methods of preparing the adsorbed layers of polymers would suggest that, in order to obtain effects on the turbulent drag

of turbulent shear flow, an adsorbed-entangled layer is necessary. The inconsistency of the effects of the adsorbed layers in turbulent shear flow frustrated our aim of using the laser-doppler anemometer to study the effects of these layers on the structure of a turbulent shear flow.

However, our results, above, showed a consistent effect of the adsorbed layers of Polyox WSR-301 in increasing the total numbers of oscillations of a water column in a flexible PVC tube. The adsorbed layers showed, also, a consistent effect in reducing the value of the damping constant of a water column oscillating freely in a coated flexible PVC tube relative to its oscillation in a clean flexible PVC tube. Our analysis above would indicate that the effects of the adsorbed layers on the damping of the liquid column is associated with improving the wetting of the surface of the PVC tube rather than with inducing changes in the structure of the bulk flow.

In the field of the free oscillation of the liquid, our results on the damping of a water column agreed with the published results of previous investigators (Ury (1962), Biery (1963), Park and Baird (1970) and McComb (1974)). Our results on the damping of a water column in a flexible PVC tube combined with the reported results of Ury (1962) and Park and Baird (1970), on the damping of a mercury column in plastic and glass tubes respectively, would indicate that Ury's calculations of the damping constant of a freely oscillating liquid column represent an upper bound. (The effects of the surface tension and the secondary flow on the damping of the liquid column included.) Our results on the damping constant of a water column in a glass tube coupled with the results of Biery (1963) and Park and Baird (1970) would indicate that Biery's calculations (1969) represent a lower bound. (The surface

tension and the secondary flow effects neglected.) Our results coupled with the results of other researchers would indicate that the surface tension effects on the damping of a liquid column depend on the combined characteristics of the liquid and the tube surface.

Our results on the effects of the drag-reducing polymers and Aerosol additives would indicate that these additives affect the wetting of the tube surface and possibly the secondary flow associated with the liquid column oscillation. Changes in the structure of the bulk flow, outside those associated with the increase in the viscosity due to the additives, seem unlikely. No changes in the frequency of oscillation of the liquid column due to the additives were observed. No anomalous results in the damping of the liquid column, that may be attributed to the visco-elasticity of the test solutions, were observed. A possible explanation of the absence of any clear effects of the visco-elastic characteristics of the polymer solutions on the damping of liquid column oscillation may be attributed to the fact that the time constant of the liquid column oscillation is very large in comparison with the relaxation time of the test drag-reducing polymers. Drag-reducing polymers tend to exhibit their anomalous effects when their relaxation time is of the same order of magnitude or larger than the time constant of the flow (Little et al (1975)). Our analysis in Chapter 3 seems to support this hypothesis. According to the reported results of Biery (1964), Seyer and Metzner (1967) and Everage and Gordon (1971), the relaxation time of the test polymer solutions lie in the range $2 \times 10^{-2} - 3 \times 10^{-4}$ sec. depending on the molecular weight of the polymer and the concentration of the solution. These values are very small in comparison with the time constant of the test liquid column oscillation of 1.7 - 1.8 sec. Consequently, according to the

hypothesis of the time-scale effect the polymer additives would not exhibit any clear effects on the dynamics of liquid column oscillation of such time constants.

It seems to be a formidable task to check the validity of the time-scale hypothesis by using the free oscillation of the liquid column. Such a method requires that the length of the liquid column be much less than one millimetre, in order to obtain a time constant of the flow field comparable with the available data on the relaxation times of polymer solutions. However, a forced oscillating liquid column seems a likely method.

First Run	Second Run	Third Run	Fourth Run	Fifth Run	Average Peak
17.1	17	17.1	16.9	17	17
12.8	12.8	12.9	12.8	12.7	12.8
9.2	9.2	9.3	9.2	9.1	9.2
7.2	7.3	7.2	7.1	7.2	7.3
5.3	5.3	5.3	5.2	5.3	5.3
4.3	4.2	4.2	4.3	4.2	4.2
3.1	3.1	3.2	3.1	3.1	3.1
2.5	2.4	2.4	2.3	2.4	2.4
1.7	1.7	1.7	1.6	1.7	1.7
1.2	1.1	1.2	1.2	1.2	1.2

Table 4.1

Amplitudes of successive oscillations of a free oscillating liquid column

No. of the Half Cycle	Damping Constant
1	.0900
2	.0973
3	.0908
4	.0924
5	.0887
6	.0900
7	.0887
8	.0913
9	.0934
10	.1011

First six half cycles:

Average damping constant = .0913

% Std Deviation = 3.2

Ten half cycles:

Average damping constant = .0922

% Std Deviation = 4.19

N.B. Data of the amplitudes of successive oscillation is that of Table 4.1.

Table 4.2

Damping constant per half cycle of liquid column oscillation

CHAPTER 5

MEASUREMENTS OF POLYMER EFFECTS ON FLUID PROPERTIES RELEVANT TO BUBBLE DYNAMICS

5.1 INTRODUCTION

In preceeding chapters we discussed some aspects of bubble dynamics in Newtonian and non-Newtonian liquids. The mechanisms of energy losses by small-amplitude bubble pulsation were also investigated, namely the viscous, the acoustic radiation and the thermal losses. It was shown that the thermal losses of a pulsating bubble are essentially controlled by the physical properties of the gas in the bubble, while the acoustic and viscous losses are mainly dependent on the physical properties of the liquid.

For a Newtonian liquid (Chapter 2) the acoustic radiation damping factor may be expressed as

$$b_{ac} = \frac{\rho\omega^2}{4\pi c} \tag{5.1}$$

ρ, the density, is a physical property of the liquid. C, the velocity of sound in the liquid, depends on the physical properties of the liquid, the density and the bulk modulus. Any change in one or both of these properties would affect the acoustic damping factor unless such changes cancel the effects of each other.

In a real liquid, the viscous dissipative term is expressed (Chapter 2) as

$$b_{vis} = -3 \int_{R_o}^{\infty} \frac{\sigma_{rr}}{r}\, dr \tag{5.2}$$

In both Newtonian and non-Newtonian liquids σ_{rr}, the deviatoric viscous

stress depends on the viscosity of the solution among other factors. The viscosity is a property of the liquid. Any change in the viscosity due to the addition of polymer solutions would result in changes in the viscous damping factor.

Being relevant to the damping of pulsating bubbles, the velocity of sound in and the viscosity of the polymer solutions, of concentrations of interest, were determined experimentally. The aim of this Chapter is to present an account of the work done in this field.

5.2 VELOCITY OF SOUND IN DRAG-REDUCING POLYMER SOLUTIONS

The velocity of sound in drag-reducing polymer solutions has received very little attention. The author can find no extensive or comprehensive treatment of this matter. However, there are some reports on the propagation of sound in concentrated polymer solutions. Some of these polymers are known as drag-reducers (Polymethylmethacrylate, PMMA) Velocity of sound in liquids possessing viscous and elastic properties has been studied mainly in connection with the propagation of ultrasonic sound waves. Pryor (1954) has studied the absorption of ultrasonic sound waves (4-18 MHz) in highly concentrated solutions of rubber in benzene and perspex in pyridine. Pryor reported that in a 10% rubber solution in benzene, the velocity of sound is 1335 m/s, while in pure benzene it is 1325 m/sec. No results on the velocity of sound in perspex-pyridine solutions were reported. Bader and Cerf (1970) measured the velocity of ultrasonic waves (1.5 - 11 MHz) in concentrated solutions of Polystyrene (PS), Polymethylmethacrylate (PMMA) and Polyvinylisobutylether (PUP) in tolouene, benzene and water. Their reported results were taken at a frequency of 1.5 MHz and showed, in most cases, a change of less than 2%.in the velocity of sound due to the addition of polymers.

Below, we will present a description of the experimental set-up and the experimental results of the velocity of sound in aqueous polymer solutions of Separan AP-273 and Polyox WSR-301.

5.2.1 Experimental Set-Up

In measuring the velocity of sound we used the standing wave method. In this method, a train of waves of fixed frequency are sent through a liquid column in a tube. The waves would be reflected at the surface of the liquid column or at a reflecter immersed in the liquid column. When reflected, the waves travel in the opposite direction. Standing waves would thus be formed as a result of the interaction between the oppositely travelling waves. Standing waves have steady min/max called nodes and anti-nodes. The distance between two successive nodes or anti-nodes is the length of the standing waves. The length of a standing wave is half that of a travelling one at the same frequency. The velocity of sound is

$$c = f \times \lambda_{tr} \tag{5.3}$$

where

c = velocity of sound

f = frequency of acoustic waves

λ_{tr} = travelling wave length.

Since the length of a travelling wave is twice that of a standing wave, the velocity of sound thus becomes

$$c = f \times 2\lambda_{st} \tag{5.4}$$

where λ_{st} is the length of the standing wave.

A schematic diagram of the standing wave apparatus is shown in Figure (5.1). A steel tube 226 cm long, 3.74 cm ID and 4.78 cm OD was clamped to a wall and insulated from external vibrations by rubber vibration absorbing insulators. The tube was flanged at the lower end and a rubber diaphragm was fixed there. A water-tap was fixed very close to the lower end of the tube for drainage purposes. The upper end of the tube was opened to the atmosphere. An over-flow connection was fixed close to the upper end to maintain the water column at fixed length. A holder was fixed at the upper end of the tube to hold the sensing element used to measure the pressure amplitude of the standing wave.

The sensing element consisted of a couple of transducers held together in a prespex holder. A connection from the transducers was taken to the electronics. The connection was led through a .625 cm diameter copper tube fixed to the prespex holder. The copper tube passed through the holder at the upper end of the tube.

The train of acoustic waves passing through the liquid column was generated by a vibrator fixed to the rubber diaphragm at the lower end. The vibrator (Lings Dynamic Systems, Series 100) has an operating frequency range of up to 12 KHz. The vibrator was driven by a power oscillator (Goodmans Industries Ltd).

The output of the sensing element was connected to a charge amplifier whose output is displayed on a two-channel oscilloscope (Gould OS 4000). The power terminals of the vibrator were also connected to the oscilloscope in order to check the consistency of the input and output frequencies. A digital frequency meter

(Advance Instruments TC 11A) displayed the input or output frequency. The output signal was also connected to a RMS voltmeter (DISA 55D35) then to a digital voltmeter (Solartron LM 1420.2) where the pressure amplitude of the standing wave was read.

5.2.2 Testing of Apparatus

Before using polymer solutions, the apparatus was tested using water, because the velocity of sound in water is well-established. Two frequencies were chosen as testing frequencies. The sensing element was moved along the water column in steps, and the digital voltmeter output noted. The collected data were then plotted and the length of the standing wave calculated. The experimental results of the tests for both frequencies are listed in Table 5.1. Representative graphs of standing waves are presented in Figures (5.2) and (5.3).

The velocity of sound in water at 15°C is 1460 m/sec (Woods (1964)). Using the standing wave method, Woods obtained a result of 1490 m/sec. At a frequency of 1450 Hz the travelling wave length is 100 cm. A standing wave of the same frequency would be 50 cm long. For a frequency of 2100 Hz the wave lengths are 69 cm and 34.5 cm respectively. As shown in Table 5.1, the experimental results are in good agreement with the theoretical ones. The small deviation of the experimental results from the theoretical ones is probably caused by the accuracy of the measurements in determining the anti-nodes of the standing wave, the purity of the water (presence of suspensions or dissolved air) and the attenuation of the sound waves due to the viscosity of water.

5.2.3 Effect of Drag-Reducing Polymers on the Velocity of Sound

The velocity of sound was measured in two polymer solutions; Separan AP-273 and Polyox WSR-301 in a range of concentrations from 50-3000 ppm at a frequency of 1450 Hz. This frequency was chosen because it lies in the range of resonance frequencies of the bubbles of interest.

The experimental results are listed in Table 5.2. The results show that the velocity of sound in drag-reducing polymer solutions is the same as that in water within experimental error.

However, although our experimental results showed no change in the velocity of sound due to the addition of drag-reducing polymers, a conclusive judgement, as to the possible effects of these polymers on the velocity of sound, cannot be based solely on our experimental results. Such a conclusive judgement could only be based on more extensive investigation covering different forms of acoustic waves, different methods of sound velocity measurements and a wide range of frequencies, very low to very high.

5.3 VISCOSITY MEASUREMENTS OF DRAG-REDUCING POLYMER SOLUTIONS

Equation (5.2) represents the viscous dissipation term in the equation of bubble dynamics in a real liquid. For the case of a Newtonian liquid, the deviatoric stress component is related to the strain rate through the relationship

$$\sigma_{ij} = 2\mu e_{ij} \tag{5.5}$$

where μ is the dynamic viscosity of the Newtonian liquid and is dependent on the temperature only. In the literature, different types of viscosities are in circulation and are defined depending on the geometry

of the flow; shear viscosity, extensional or elongational viscosity and volumetric viscosity. By viscosity, when referring to Newtonian liquids, we mean the dynamic viscosity as defined in Equation (5.5). The dynamic viscosity is numerically equal to the shear viscosity.

In non-Newtonian liquids, Equation (5.5) no longer holds and the viscosity becomes dependent on the strain rate in addition to the temperature. The viscosity of non-Newtonian liquids may either increase or decrease with changes in the strain rate. If the viscosity increases as the strain rate increases the liquid is known as a shear thickening (Dilatant). If the viscosity decreases the liquid is a shear thinning (Pseudo-plastic). Drag-reducing polymer solutions of interest are shear thinning liquids whose viscosity decreases with increases in the strain rate. At a fixed strain rate, however, the viscosity of the polymer solution increases the higher the concentration of the polymer in the solution.

The viscosities of two polymer solutions of interest; Separan AP-273 and Polyox WSR-301, were measured using a Ubbelohde viscometer (see Figure (5.4)). In this viscometer, the viscosity of a liquid is measured relative to that of a Newtonian liquid whose viscosity is known. The viscosities of the above mentioned polymer solutions were measured relative to that of water. The viscosity of water is well-documented and at $20^{\circ}C$ it has a value of 1 cp ($.01 \frac{gm}{cm.sec}$).

In measuring the viscosity of a liquid in a Ubbelohde viscometer the liquid is pushed from bulb A to a point above bulb B while C is kept closed. C is then opened and the liquid allowed to flow down tube D. By measuring the time taken by the liquid to flow from point X to point Y, the viscosity of the liquid could be determined through the relationship

$$\mu_{solution} = \mu_{solvent} \times \frac{T_{solution}}{T_{solvent}}$$

where the subscript 'solvent' refers to a Newtonian liquid whose viscosity is known.

The experimental results of viscosity measurements are presented in Table 5.3 and are plotted in Figure (5.5). Viscosity measurements were taken at a strain rate of 2500 sec^{-1}. The experimental results of viscosity measurements will be used in the calculations later to determine the would-be damping constant in a Newtonian liquid of equivalent viscosity. In doing that we take into consideration that the viscosity of polymer solutions is strain rate dependent.

According to Equation (2.38) the strain rate of a pulsating bubble is

$$e_{rr} = \frac{\partial}{\partial r}\left(\frac{R^2R'}{r^2}\right)$$

At the bubble surface, the linearised strain rate is

$$e_{rr} = -2\frac{R'}{R_o} \tag{5.6}$$

The strain rate could be calculated from Equation (5.6) when the velocity, $\dot{R}$, is known. The solution to the equation of bubble dynamics is terms of the change in the radius, ΔR, is

$$\Delta R = \Delta R_o \exp(-\xi \pi f_o t) \sin(\omega_o t) \tag{5.7}$$

where

ΔR_o = maximum change in bubble radius.

Differentiating both sides of Equation (5.7)

$$\Delta\dot{R} = \Delta R_o \exp(-\xi\pi f_o t)(\omega_o \cos \omega_o t - \xi\pi f_o \sin \omega_o t) \tag{5.8}$$

The maximum value of $\Delta\dot{R}$ is

$$\Delta\dot{R} = \Delta R_o \omega_o \tag{5.9}$$

ΔR is positive or negative depending on the direction of the velocity, ie. compression or expansion.

Substituting Equation (5.9) in Equation (5.6) we obtain

$$e_{rr_{max}} = 2\,\omega_o\,\frac{\Delta R_o}{R_o} \tag{5.10}$$

The bubbles of interest in this work pulsated at an angular frequency of about 7000 rad/sec.

$$e_{rr_{max}} = 14{,}000\,\frac{\Delta R_o}{R_o} \tag{5.11}$$

No absolute measurements of the change in the bubble radius were made in this work. In the table below, we present some calculated values of the maximum strain rate at different ratios of $\frac{\Delta R_o}{R_o}$

$\frac{\Delta R_o}{R_o}$	Strain rate sec^{-1}
0.001	14
0.01	140
0.1	1400

The strain rate at $\frac{\Delta R_o}{R_o} = 0.1$ is of the same order of magnitude as that at which our viscosity measurements were made. However, Strasberg (1956) calculated the velocity of the bubble wall using frames of motion-picture. The volume of the bubble he investigated was 69 mm^3. The bubbles of interest in our work had a volume of 110 mm^3. Strasberg found that

$$\frac{3\Delta R}{\Delta t R} = 45 \quad sec^{-1}$$

Taking the above data of Strasberg and applying it to the

bubbles of interest in this work we obtain

$$\Delta R_o = 15 \times 3 \times 10^{-1} \times \frac{10^{-3}}{1.1}$$

$$= 4 \times 10^{-3} \text{ cm}$$

substituting this value in Equation (5.11) we obtain

$$e_{rr_{max}} = 14{,}000 \times \frac{4 \times 10^{-3}}{0.3} = 190 \text{ sec}^{-1}$$

According to the above calculations the maximum strain rate of the pulsating bubble is an order of magnitude less than the rate of strain at which the viscosity of polymer solutions were measured. A fuller account of the effect of the strain rate on the viscosity of the polymer solutions and the behaviour of the pulsating bubble will be presented in a later chapter.

Polymer solutions are reported to exhibit high resistance to stretching and consequently have high extensional viscosity. The extensional viscosity of a Newtonian liquid, as defined in the literature (Astarita & Nicodemo (1971)), has a value three times that of the dynamic viscosity. In polymer solutions this value could become orders of magnitude higher depending on the strain rate.

The experimental results of the damping constants of pulsating bubbles in polymer solutions will be compared with the damping constants of the same bubbles in Newtonian liquids of equivalent viscosity. This will allow us to assess the effects of elasticity in the polymer solutions on the pulsation of bubbles.

Frequency Hz	Theoretical Stdg wave length/cm	Measured Stdg wave length/cm	Frequency Hz	Theoretical Stdg wave length/cm	Measured Stdg wave length/cm
1450	50	51	2100	34.5	35
1450	50	52	2100	34.5	34
1450	50	50.5	2100	34.5	34
Mean length		51.17	Mean length		34.33
Std Deviation σ		.624	Std Deviation σ		.47
% $\frac{\sigma}{L_m}$		1.22	% $\frac{\sigma}{L_m}$		1.37

Table 5.1

Testing Results of Sound Velocity Measurements in Water

Solution	Measured Velocity m/s	Standard Deviation	% Standard Deviation
Water	1484	.62	1.22
Polyox WSR-201			
50 ppm	1479	.71	1.39
100	1465	.41	0.81
300	1474	.62	1.22
600	1456	.85	1.70
1000	1450	.67	1.33
3000	1450	1.41	.82
Separan AP-273			
50 ppm	1470	.47	.93
100	1479	.82	1.60
300	1479	.41	.81
600	1450	.82	1.60
1000	1470	.47	.93
3000	1513	.24	.45

Table 5.2

Experimental Results of the Velocity of Sound in water and polymer solutions

Concentration ppm	Polyox WSR-301 Viscosity cp	Separan AP-273 Viscosity cp
10	1.01	1.01
20	1.021	1.038
30	1.025	1.065
50	1.053	1.15
100	1.103	1.333
200	1.243	1.81
250	1.334	2.24
300	1.372	2.62
400	1.548	3.56
500	1.728	4.72
600	1.898	6.12
800	2.28	9.00
1000	2.795	13.52
1500	4.59	24.8
2000	9.83	50.1
3000	12.022	-
4000	23.596	-
5000	43.024	-
6000	80.90	-

Table 5.3

Measured viscosity of Polymer solutions

Temperature = 20°C

Strain-rate = 2500 sec^{1}

CHAPTER 6

DESIGN AND TESTING OF EXPERIMENTAL SET-UP

6.1 INTRODUCTION

According to the linear theory of bubble pulsation in liquids, the resonance frequency of a bubble, experiencing no energy losses (Minnaert (1933)) is

$$f_o = \frac{1}{R_o}\left[\frac{3\gamma P_\infty}{\rho}\right]^{\frac{1}{2}} \qquad (6.1)$$

The notation is the same as in previous chapters.

It could be seen from Equation (6.1) that the resonance frequency of a bubble is inversely proportional to its equilibrium radius. It was shown earlier (Chapter 2) that the different mechanisms of energy loss (viscous, acoustic and thermal) all depend on the frequency or the equilibrium radius of the bubble, among other factors. Therefore, it becomes necessary, when studying the dynamic behaviour of bubbles, to produce bubbles of known sizes and frequencies.

In order to comply with such requirements an apparatus was designed and tested. Below we present a detailed description of the experimental set-up and the bubble-generating apparatus. The results of the tests will also be presented.

6.2 DESIGN OF THE EXPERIMENTAL SET-UP

A schematic diagram of the experimental set-up is shown in Figure (6.1). It consisted of a bubble generation apparatus connected to a nozzle submerged in a liquid contained in a steel tank. A hydrophone, submerged in the liquid, was situated at a distance from the outlet of the nozzle. The hydrophone was connected to a frequency analyser. The

output from the frequency analyser can then be displayed on recorded as appropriate.

The bubbles were formed by compressing air through the submerged nozzle. Once the bubble was detached it pulsated and produced acoustic waves. The hydrophone picked up the acoustic pressure wave and passed it to the frequency analyser. The output from the frequency analyser was monitored as appropriate.

A cross-sectional view of the air supply system is shown in Figure (6.2). It consisted of an electric motor, a piston and a cylinder, and was fixed on a wooden base. The cylinder consisted of two parts: the cylinder body and the cylinder head. The cylinder body was 12 cm long and was made from brass. It had an internal diameter of 1.74 cm and an outside diameter of 2.5 cm. The outlet from the cylinder consisted of a protruding nozzle .25 cm ID x .57 cm OD. The outlet was connected to the submerged nozzle by a flexible PVC tube .47 cm ID x .96 cm OD. The cylinder body had a rectangular groove on its surface. The cylinder head was made of a circular brass block 2.5 cm diameter x 2 cm thick and connected to the cylinder body by four brass bolts. A threaded hole was drilled in the cylinder head through which a threaded connecting rod passed. The connecting rod joined the electric motor with the piston inside the cylinder body. The cylinder passed through a circular hole in a steel block which acted as a holder. A threaded hole was drilled through the steel block and a bolt passed through the hole down to the groove on the surface of the cylinder body.

The piston was also made from brass. It was 2.8 cm thick and had two O-rings to ensure maximum tightness and to prevent any air leakage. The piston was connected to the electric motor by the threaded connecting

rod. The electric motor was single phase operating on 240 volt power supply running at 1450 rpm. The speed of the motor was reduced through a speed-reduction gear to 3 rpm.

In the apparatus described, the air was pushed out by the backward sliding motion of the cylinder rather than the piston. The rotation of the motor spindle rotated the connecting rod which in turn rotated the piston. The connecting rod rotated the cylinder too but because the cylinder could not rotate because of the steel bolt passing through the steel block to the groove on the surface of the cylinder, the cylinder itself was forced to slide back. The cylinder slid back while the piston was held fixed and the air was pushed out through the outlet to the submerged nozzle. Bubbles were thus formed.

The test nozzle was submerged in water in a steel tank 75 x 60 x 70 cm. The nozzle was held by a sponge bush fitted to a brass holder. The steel tank was mounted on an expanded polystyrene sheet, 5 cm thick, on a concrete slab. The polystyrene sheet acted as a vibration insulator to prevent the transmission of external vibration thus reducing the effects of background noise. Nozzles of different materials were used in the preliminary tests and as a result it was decided to use brass nozzles. Brass nozzles are easily machined to the required sizes and are not affected by water or polymer solutions.

To check the uniformity of bubble production, nozzles of different sizes were used. All the nozzles were 7.5 cm long but varied in diameter between 0.2 - 0.5 cm. The results of bubble uniformity production are plotted in Figure (6.3) where it is shown that bubbles are produced more uniformly with a 0.5 cm diameter nozzle. The 0.5 cm nozzle was used throughout most of the experiments. As a later stage in the work, nozzles of other materials were used and in particular flexible PVC

nozzles. In a later chapter we will discuss in detail why nozzles of different materials were used.

The acoustic pressure caused by the pulsation of the bubble was picked up by a Bruel & Kjaer general purpose hydrophone Type 8101. The hydrophone had a piezo-electric ceramic sensing element well-shielded from the electronic noise and was mounted in a sound transparent boot of neoprene rubber. The acoustic impedance of the neoprene rubber approaches that of water. The sound pressure was thus coupled from the water to the sensing element with good efficiency. The hydrophone was 24.8 cm. long and 2.4 cm diameter. It has a uniform frequency response between 1 Hz and 60 KHz. The hydrophone could operate in a temperature range - $10^{\circ}C$ to $65^{\circ}C$ and under a static pressure of 40 atmospheres.

The hydrophone was connected to a Bruel & Kjaer frequency analyser Type 2107. The analyser was of the constant percentage band width type with a linear frequency response from 2 Hz to 40 KHz. The output of the analyser could then be processed as appropriate.

A storage type oscilloscope (Gould OS4000) was used to store the signal of the bubble pulsation. Making use of the time scale of the oscilloscope, the frequency of the bubble could be determined. The damping constant of bubble pulsation could also be determined by finding out the ratio of the heights of successive amplitudes. A ramp output from an oscilloscope (Type OS250) was connected to a Time Counter (Type TC 11A). The Time Counter was triggered on and off by the signals of pulsation of successive bubbles. In this way it was possible to determine the length of time required to produce a bubble and consequently the uniformity of bubble production.

An oscillograph (Bell and Howell model 5-137) was used to produce prints of the signal of bubble pulsation so that the necessary analysis could be carried out.

A Racal-Thermionic tape recorder was also used to record the signal and digitise it by using an A-D converter, then carrying out the necessary analysis. The tape recorder had four speed $1\frac{3}{4}$ - 15 in/sec and a linear frequency response of 2.5 KHz which was well above the range of frequencies of interest.

6.3 TESTING OF APPARATUS

The apparatus was designed with the intention that its uniform speed and uniform volume displacement would result in producing uniform air bubbles. The first step to test the apparatus was to check the uniformity of the speed of sliding of the cylinder. Two points 40 mm apart were marked at the surface of the cylinder. The time taken by the cylinder to travel this distance was measured. The results of five runs were taken and are listed in Table 6.1. Knowing the distance and the time, the average speed and its standard deviation were calculated and are also listed in Table 6.1. The results proved satisfactory, showing a uniform running of the apparatus.

The next step was to check the conformity of the volume of air displaced by the piston to the one calculated from the dimensions and speed of the cylinder. A glass cylinder was inverted above the submerged nozzle outlet where the escaping bubbles collected. The volume of collected air was measured directly from the graduations on the glass cylinder surface. The results of three runs were taken and, as above, the travelled distance and time were both noted in order to double check the validity of the previous results. The previous results of

the speed of the cylinder were confirmed as shown by the time taken to cover the 40 mm distance listed in Table 6.2. The calculated displacement volume was found out by multiplying the internal cross-sectional area of the cylinder by the travelled distance. The results of the test are listed in Table 6.2. The discrepancy between the calculated and actual displaced volumes is well within experimental error.

At this stage of the testing procedure, the apparatus proved satisfactory in terms of uniform speed and volume displacements. The next step was to find out the average volume of the bubbles. In calculating the average volume of the bubbles, a certain number of bubbles were collected in the glass cylinder. The time required to produce that same number of bubbles was measured. By dividing the collected volume of air by the number of bubbles, the experimental average bubble volume was found. Also, by knowing the time taken to produce the bubbles and the average speed of the cylinder, the average bubble volume was calculated. Results of this test are listed in Table 6.3 and were satisfactory.

The last stage of the testing procedure was to check the uniformity of the bubble production, and in turn, the uniformity of the size of the bubbles produced. The uniformity of bubble production was checked by noting the time duration taken to produce a single bubble. On one of the oscilloscopes a ramp output was connected to a time counter where the latter was triggered whenever a signal appeared on the former. A first bubble signal would trigger the counter-timer on while the next signal would trigger it off. The time duration between the two signals could then be read directly on the screen of the time-counter, and represents the time required by the apparatus to produce a single bubble.

The volume and radius of the bubble may then be calculated since the required parameters for this calculation are all known. The results of a typical run are listed in Table 6.4. The calculated frequency of single bubbles are also listed in Table 6.4. The bubble frequency was calculated using Equation (6.1).

To check the agreement of the theoretical bubble frequency with the experimental one, the latter was measured by a spectrum analysis of the bubble signal. The analysis was done using the frequency analyser and a storage oscilloscope. The frequency analyser had a tuned filter that could be tuned to any frequency within its operating range (2 - 40,000 Hz). The filter was tuned over a range of frequencies that overlap the theoretically predicted frequency of the bubble. The amplitude of the bubble noise was measured at different frequencies and the results plotted. The resonance frequency of the bubble is that at which the bubble will have the highest noise amplitude. Figure (6.4) shows a typical graph of spectrum analysis of a bubble signal.

Other methods were used to measure the resonance frequency of the bubble. Among them was one that depended on measuring the time intervals between successive peaks of a bubble signal traced on the storage oscilloscope. By measuring the time between successive signal peaks the bubble frequency was found. Another method was developed in the course of computational analysis of the bubble signal. The signal was digitised at a known sampling rate. The number of samples between successive peaks of the signal were calculated. By dividing the sampling rate per second by the number of samples per cycle the resonance frequency was calculated.

Distance/mm	Time/sec	Average Speed mm/sec
40	830	.0482
40	835	.0479
40	834	.0480
40	829	.0483
40	832	.0481
Mean speed Vm		.0481
Standard Deviation σ		.00014
% $\frac{\sigma}{Vm}$		.29

Table 6.1

Testing Results of Bubble Generation Apparatus Speed

Calculated volume mm^3	Time/sec	Actual collected volume/mm^3
9510	829	9480
9510	832	9560
9510	834	9440
Mean volume Vm		9493
Standard Deviation σ		49.89
% $\frac{\sigma}{Vm}$		0.53

Table 6.2

Testing Results of Apparatus Volume Displacement

Calculated volume/mm^3	Collected volume/mm^3	No of Bubbles	Average volume of bubble
5700	5550	50	111.0
5600	5580	50	111.6
5640	5620	50	112.4
Mean Bubble Volume Vm			111.67
Standard Deviation σ			.57
$\% \frac{\sigma}{Vm}$			.51

Table 6.3

Testing Results of Bubble's Average Volume

Time sec	Volume mm^3	Radius mm	Frequency Hz
10.2	116.52	3.03	1077.17
10.0	114.23	3.01	1084.31
9.9	113.09	3.00	1087.95
9.7	110.81	2.98	1095.37
9.8	111.95	2.99	1091.63
9.6	109.66	2.97	1099.16
9.7	110.81	2.98	1095.37
9.6	109.66	2.97	1099.16
9.8	111.95	2.99	1091.63
9.7	110.81	2.98	1095.37
9.5	108.52	2.96	1103.01
9.6	109.66	2.97	1099.16
9.7	110.81	2.98	1095.37
9.6	109.66	2.97	1099.16
9.5	108.52	2.96	1103.01
9.3	106.24	2.94	1110.86
9.6	109.66	2.97	1099.16
9.4	107.38	2.95	1106.90
9.7	110.81	2.98	1095.37
9.4	107.38	2.95	1106.90
9.2	105.10	2.93	1114.87
9.4	107.38	2.95	1106.90
9.3	106.24	2.94	1110.86
9.5	108.52	2.96	1103.01
9.2	105.10	2.93	1114.87
9.4	107.38	2.95	1106.90
9.2	105.10	2.93	1114.87
9.2	105.10	2.93	1114.87
9.2	105.10	2.93	1114.87
9.4	107.38	2.95	1106.90
9.1	103.95	2.92	1118.94
9.1	103.95	2.92	1118.94
9.5	108.52	2.96	1103.01
9.2	105.10	2.93	1114.87
9.4	107.38	2.95	1106.90
9.2	105.10	2.93	1114.87
9.2	105.10	2.93	1114.87
9.2	105.10	2.93	1114.87
9.1	102.81	2.91	1123.06
9.4	107.38	2.95	1106.90
9.1	102.81	2.91	1123.06
9.4	107.38	2.95	1106.90
9.0	102.81	2.91	1123.06
9.0	102.81	2.91	1123.06
9.2	105.10	2.93	1114.87
9.0	102.81	2.91	1123.06
9.0	102.81	2.91	1123.06
9.3	106.24	2.94	1110.86
9.1	103.95	2.92	1118.94
9.2	105.10	2.93	1114.87
Mean Value	9.4	107.33	2.95	1107.28
% Std Deviation	0.030	0.030	0.010	.00988

Table 6.4

Testing Results of Uniformity of Size and Frequency of Bubbles

CHAPTER 7

EXPERIMENTAL PROCEDURE

In this chapter, we present a detailed description of the experimental procedure followed in the experimental work on bubble pulsations. The main aim of the procedure was to obtain, as accurately as possible, the damping constant of bubble pulsation in water and drag-reducing polymer solutions. We will also present a description of the polymers used in the experimental work, their properties and the method of preparing their aqueous solutions.

7.1 DETERMINATION OF THE DAMPING CONSTANT OF BUBBLE PULSATION

Devin (1959) documented the different methods used by different researchers to determine the damping constant of bubble pulsation. These were the successive oscillation method, the width of the resonance method, the standing-wave ratio method and the resonance absorption method. In this work we used the method of successive oscillations because it is the only method available for measuring the damping constant of free pulsating bubbles. The other methods are suitable for forced oscillation. This method is appropriate for relatively large bubbles because the parameters of the bubble signal can be measured with high accuracy. For small bubbles, this method becomes less accurate.

In the successive oscillations method, the damping constant is calculated from the logarithmic decrement of the successive peaks of a bubble signal. The bubble signal is picked up by a hydrophone submerged in the liquid at a distance from the bubble. The signal is amplified and displayed on the screen of a storage oscilloscope.

Prints of the signal can be obtained on oscillograph paper.

The signal of a freely pulsating bubble is a damped sinusoidal signal described by the equation

$$\ddot{A} + 2\zeta\omega_o^2 \dot{A} + \omega_o^2 A = 0 \qquad (7.1)$$

The solution of Equation (7.1) in terms of ζ is

$$\frac{\zeta^2}{1 - \zeta^2} = \left[\frac{1}{n\pi} \ln \frac{A_o}{A_n}\right]^2 \qquad (7.2)$$

If $\zeta << 1$, Equation (7.2) becomes

$$\zeta = \frac{1}{n\pi} \ln \frac{A_o}{A_n} \qquad (7.3)$$

where

A_o = amplitude of pulsation of the initial peak

A_n = amplitude of pulsation of the n^{th} peak

n = number of peaks

ζ = damping constant of bubble pulsation.

The damping constant, ζ, can be easily calculated if the values of the peaks of the bubble signal are known. A trace of the bubble signal on the screen of a storage oscilloscope or on the graph paper of an oscillograph can be easily used for this purpose. However, the accuracy of the successive oscillation method using traces of bubble signal is not so high because of the errors involved in measuring the exact values of the peaks of the signal. The method of successive oscillations was used by Bauer (unpublished results reported by Strasberg (1956) and Devin (1959), Koger and Houghton (1968) and McComb and Ayyash (1976)). In the above works, the damping constant was calculated using traces of the bubble signal on an oscilloscope or an oscillograph.

In the early stages of this work, the damping constant was calculated by obtaining prints of the bubble pulsation signal, and then carrying out the necessary calculations as outlined above. This procedure was cumbersome and it was not feasible for taking a number of bubbles, averaging them and finding their standard deviation. Consequently, a more accurate procedure was developed and used to calculate the damping constant. In the new procedure, the bubble signal was recorded on a tape recorder and then inputted to an A/D Converter. The signal, then in a digital form, was processed on the computing facilities of the Edinburgh Regional Computing Centre (ERCC).

7.2 DIGITISATION OF THE BUBBLE SIGNAL

The bubble signals were recorded on a Racal-Thermionic tape recorder. The recording speed was 15 in/sec. A number of signals (about 15 signals) were recorded in each run. The bubbles were produced uniformly (see Chapter 6) and so were the signals. Consequently, we did not find it necessary to record larger numbers of bubble signals. The damping constants of single bubbles are listed in Table 7.1 where it is shown that the change in the damping constants of single bubbles is very small. Koger and Houghton (1968) reported that they superimposed 10 - 15 bubble signals on the screen of an oscilloscope from which they calculated the damping constant of the average bubble.

The digitisation of the bubble signals was carried out on the A/D Converter of a PDP-15 machine running a continuous Analogue Data Collection program (ANACOL). The program collected analogue data from the A/D Converter and stored it in a magnetic tape file. The file consisted of data buffers each containing 500 samples. The number

of buffers in a file could be anything up to a four integer number. The timing of the data collection was controlled by a triggering signal. The voltage input to the A/D Converter was in the range ± 5 volts as recommended. In integer terms, an input of ± 5 volts corresponded to values in the range - 2048 to 2047. In running the program, the number of channels, the number of samples per data buffer and the number of buffers required were all inputted as program parameters.

While the data was being collected it was monitored on a video screen. The readings from the A/D Converter were plotted across the left hand section of the screen and ranged between ± 5 volts from top to bottom. The picture drawn was a moving graph with the most recent points at the right.

The maximum sampling rate of the machine was just over 5000 samples per second. For a frequency of about 1100 Hz, this sampling rate gave about 5 samples per cycle. In order to increase the sampling rate the tape recorder was played back at 1/8 the recording speed. Playing the tape recorder at this low speed gave an actual sampling rate of over 40,000 samples per sec and consequently about 40 samples per cycle. It was important to work with a high sampling rate in order to determine the peaks more precisely and thereby the damping constant.

Each bubble signal was stored in a separate file on the magnetic tape. The first few data buffers of the file contained data representing the background noise, the value of which was calculated and was taken into consideration in later calculations.

After the bubble signals were digitised and stored in data buffers on the magnetic tape, the data was transferred in the form of sequential files to the Edinburgh Multi-Access System (EMAS). The computer programs

required in the digitisation of the signal and the transfer of files to EMAS were supplied by the ERCC.

7.3 PROCESSING OF DATA

The computer programs used in processing the data on EMAS were all written in FORTRAN IV language. The data was processed as follows:

1 The value of the background noise was calculated.

2 The first 50 peaks in each signal were determined and stored in a separate files along with other parameters of interest.

3 The files containing the peaks and other parameters of different signals were concatenated. The average values of the peaks and their standard deviation were calculated.

4 The resulting average peaks represented the average bubble. The damping constant of the average bubble was calculated.

The first few data buffers in each signal contained data of the background noise and a d.c. shift introduced by the A/D Converter. The average values of the background noise and the d.c. shift were calculated and later subtracted from each of the values of the signal peaks. The average value of the background noise and the d.c. shift was about 2 - 3% of the highest peak in the signal. (For Computer Program see Appendix B.)

The next step in data processing was to determine the first 50 peaks in each signal. The computer read the data consecutively and in threesomes. Three integers, A, B and C, were examined. If B was found to be bigger than both A and C then B would be a positive peak. If B was smaller than both A and C then it would be a negative peak. If neither of the above conditions was met, then A would be dropped and a

new integer D would be introduced. The new threesome, B, C and D would then be processed as before.

In writing the program to determine the peaks, several precautions were taken. (See Appendix C.). The peaks were serialised and printed opposite to their serial numbers. The serial number of the data buffer where a peak existed was outputted. The location of the peak in the data buffer was also outputted. By knowing the location of successive peaks, the number of samples between successive peaks was calculated, and that represented a direct measure of the bubble frequency. The above parameters of each signal were stored in a separate file.

Ten files, containing the parameters of a signal, were chosen at random and concatenated. The resultant file was then processed. (For Computer Program see Appendix D.) The peaks in the ten signals were picked up and added together. The mean value of each peak of the average signal and its standard deviation was calculated. The resulting set of data was stored in a separate file. The newly created file contained the peaks of average bubble signal. The peaks were non-dimensionalised by dividing them by the highest peak value (See Appendix E.) All that was left was to calculate the damping constant of the average bubble signal.

7.4 CALCULATION OF THE DAMPING CONSTANT

As stated above, the signal of a free bubble pulsation is a damped sinusoidal one. The solution to Equation (7.1) in terms of the instantaneous amplitude, A_s, is

$$A_s = A_o \exp(-\pi\zeta f_o t) \cos 2\pi f_o t \qquad (7.4)$$

According to Equation (7.4), the amplitude is a maximum at t = 0.

In our experiments we used a frequency analyser to filter the signal and output it as appropriate. The analyser was of the constant percentage band width type. Consequently, the first few amplitudes of the signal were distorted but then they started decaying exponentially. In order to check the effect of the band width on the damping constant of the bubble signal, the band width was enlarged gradually. It was found that the width of the band has no noticeable effect on the damping constant. At large band widths, the amplitudes of the signal decay less smoothly because of the interference of other external signals.

Since the amplitudes of the bubble signal decay exponentially, then the peaks would fall on a straight line when plotted on a log-linear graph paper. The slope of the straight line represents the damping constant. In order to exclude any misjudgement or bias in plotting the best-fit straight line, the data was fitted to a best-fit straight line using the method of least squares. We chose 28 peaks (14 positive and 14 negative) out of the 50 peaks measured by previous processing of the data. The peaks excluded were the distorted ones and the last few which were distorted by the background noise. The last few peaks of any exponentially decaying signal decay at a faster rate and are highly affected by the background noise. Koger and Houghton (1968) resorted to a similar method.. They noticed that the highest amplitudes of the signal were distorted and so discarded them. Their calculation of the damping constant was based on the best exponential portion of the signal.

The envelope of a bubble signal is expressed as

$$A_s = A_o \exp(-\pi \xi f_o t) \qquad (7.5)$$

Taking the natural logarithm of both sides of Equation (7.5), there results

$$N_1 = N_2 + N_3 \tag{7.6}$$

where

$$N_1 = \ln A_s$$

$$N_2 = \ln A_o$$

$$N_3 = -\pi\delta f_o$$

By applying the least squares method (Sokolnikoff and Redheffer (1958)) the value of A_3 was calculated and hence was the value of ξ, the damping constant.

The computer program which calculated the experimental damping constant also calculated its theoretical value (see Appendix F). The components of the damping constant are (Devin (1959))

i) The thermal damping constant

$$\xi_{th} = 4.41 \times 10^{-4} f_o^{\frac{1}{2}} \text{ sec}^{\frac{1}{2}}$$

ii) The acoustic radiation damping constant

$$\xi_{ac} = \frac{\omega_o R_o}{C}$$

This component has a constant value in any liquid. The product of $\omega_o R_o$ is a constant and so is the velocity of sound C at constant temperature. In water, at room temperature, ξ_{ac} has a value of 0.014.

iii) The viscous damping

$$\xi_{vis} = \frac{8\pi\mu f_o}{3\gamma P_o}$$

In water and for bubbles larger than 0.1 cm in radius, this component is very small. For a bubble 0.3 cm in radius, the viscous damping constant

has a value of about 10^{-4}. Consequently, it was neglected in our calculations.

At a later stage in the experimental work, the digitisation facilities of the ERCC were withdrawn. Computation facilities of equivalent quality were made available. The bubble signal was inputted to an A/D Converter which fed it to a compiler connected to a video screen. The computer program which processed the signal (see Appendix G) carried out all the processes described above. The sampling rate of this computer was as high as 160000 samples per second. The computer (Computer Automation Inc., USA) proved very efficient and quick in handling the data.

7.5 PREPARATION OF DRAG-REDUCING POLYMER SOLUTIONS

In the preliminary experiments done to assess the effect of drag-reducing polymers on the damping of bubble pulsation, five different polymers were used. The five polymers belonged to two groups; the Polyox group and the Separan group.

In the Polyox group, (manufactured by Union Carbide) two polymers were used in the experiment; the polyethylene oxide (known commercially as WSR-301) and the N-3000. The former is a strong drag-reducer in turbulent shear flows, and has a very high molecular weight and is non-ionic. The latter is a weak drag-reducer in turbulent shear flows, has a relatively low molecular weight and is also non-ionic.

In the Separan group, (manufactured by DOW Chemicals), three polymers were used; AP-273, AP-30 and MG-200. The former two polymers are efficient drag-reducers in turbulent shear flows, have high molecular weights and are both anionic. The third polymer is a weak drag-reducer in turbulent shear flows, has a relatively low molecular weight

and is non-ionic. All the polymers of both groups are supplied by the manufacturers in powder form. From the results of the preliminary tests, it was decided to choose the Polyox WSR-301 and the Separan AP-273 for thorough investigation.

The Polyox WSR-301 was investigated in the concentration range of up to 6,000 wppm. The Separan AP-273 was investigated in the region up to 2,000 wppm.

In preparing the polymer solutions, the polymer was sprinkled evenly on the surface of the water in the tank and then left for some time to soak. A stirrer was then used to stir up the solution and help dissolve the polymer and mix the solution. The speed of the stirrer was regulated by a voltage regulator. The stirrer's speed was, in general, in the region of 100 rpm; well within the speed recommended by the manufacturers. The running time of the stirrer depended on the concentration of the solution; longer running time the higher the concentration of the solution.

No	Damping Constant	No	Damping Constant
1	.0285	21	.0288
2	.0285	22	.0289
3	.0290	23	.0291
4	.0286	24	.0287
5	.0283	25	.0295
6	.0286	26	.0291
7	.0287	27	.0294
8	.0287	28	.0289
9	.0288	29	.0295
10	.0287	30	.0290
11	.0289	31	.0290
12	.0290	32	.0290
13	.0291	33	.0289
14	.0284	34	.0287
15	.0291	35	.0295
16	.0287	36	.0291
17	.0293	37	.0295
18	.0291	38	.0295
19	.0285	39	.0292
20	.0287	40	.0290

Average Value = .0289

% Standard Deviation = 1.1

Table 7.1

Damping Constant of Single Bubbles

Bubble Frequency 1095 Hz

CHAPTER 8

DISCUSSION OF RESULTS

8.1 INTRODUCTION

We discussed in Chapter 4 the effects of drag-reducing polymer additives on liquid column oscillation. This chapter will be restricted to discussing the results of our investigation on the damping of bubble pulsation in Newtonian and non-Newtonian viscoelastic liquids.

Our original aim was to investigate the effects of drag-reducing polymer additives on the damping of bubble pulsation. However, it was revealed in the course of our investigation that the damping constant of bubble pulsation can be significantly affected by the geometrical and other properties of the nozzle on which the bubble is formed. This phenomenon seems to have escaped the attention of previous investigators who worked in the field of free pulsation of bubbles. This is not surprising when one recalls that free bubble pulsation has received little experimental attention. The results of our investigation of bubble pulsation are presented below in the following order:

1. The results of testing the apparatus.
2. The effects of the nozzle on the damping constant and frequency of free bubble pulsation.
3. Free damping of bubble pulsation in Newtonian viscous liquids.
4. The effects of polymer additives on the size and frequency of bubbles.
5. The effects of polymer additives on the radiation of sound by pulsating bubbles.

6. The effects of polymer additives on the damping of free pulsating bubbles.

7. The effects of NaCl additives on the damping of bubble pulsation in water and Separan solutions.

8.2 THE RESULTS OF APPARATUS TESTING

A detailed description of the bubble generation apparatus was presented in Chapter 6. Also presented were the results of the tests conducted to check the performance of the apparatus. The results of the tests are presented in Tables (6.1), (6.2), (6.3) and (6.4).

The results of the tests revealed that the apparatus ran smoothly and at a uniform speed. The collected volume of air displaced by the apparatus agreed well with the volume calculated from the dimensions of the apparatus. The uniformity of bubble production was checked by using a timer-counter. The results were satisfactory. The measurements carried out to determine the volumes of the produced bubbles showed a uniform average volume for the bubbles.

The frequency of bubble pulsation was measured by using the time base of a storage oscilloscope and traces of bubble signals on the graph papers of an oscillograph. The frequency of bubble pulsation was calculated in the course of data processing as discussed in Chapter 7. The results of measuring the frequency of the pulsating bubbles were in good agreement with Minnaert's theoretical predictions.

The bubble generation apparatus proved thus satisfactory in producing bubbles of uniform size. In fact, by measuring the length of time between successive bubbles, by using a timer-counter, it proved possible to calculate the volume, the radius and the frequency of the bubble within ± 2% of the experimental results.

8.3 THE EFFECTS OF THE NOZZLE ON THE DAMPING AND FREQUENCY OF PULSATING BUBBLES

The theory of linear bubble pulsation assumes that a gas bubble exists in an infinite mass of an incompressible fluid. The bubble is set into pulsation by an initial volume displacement or an initial velocity. The bubble then pulsates freely around its equilibrium radius. However, due to energy loss by the bubble, the amplitude of pulsation decays exponentially. The linear pulsation theory does not account for the possible effects of reverberation and scattering of the acoustic waves emitted by the pulsating bubble.

Free bubble pulsation was investigated by Strasberg (1954, 1956) Bauer (see Devin (1959)) and Koger and Houghton (1968). Strasberg (1956) investigated the emission of sound by pulsating bubbles. Bauer and Koger and Houghton (1968) investigated the damping of free pulsating bubbles. Bauer's results were up to 25% higher than the theoretical value of the damping constant. Koger and Houghton's results (1968) scattered widely around the theoretical values. No explanation was offered for the deviation of Bauer's results from the theoretical predictions. Koger and Houhgton attributed the discrepancy of their results to the high experimental error of 15% estimated for their experiments. Although the results of the damping constant deviated from the theoretical prediction, their measurements of the frequency of the bubble pulsation agreed with the theoretical predoctions of Minnaert.

No detailed description of the apparatus used by both Bauer and Koger and Houghton were given. Koger and Houghton reported using glass jets downwardly directed. A picture in Strasberg(1954) shows he used a nozzle made of a transparent material. The nozzle was situated at a downward inclination.

We used brass nozzles throughout most of the work. The nozzles were situated horizontally and held by a holder. The first holder used was made of brass. In carrying out experiments to measure the damping constant of pulsating bubbles we observed a large discrepancy between the experimental and theoretical damping constant. The experimental damping constant was higher than the theoretical one. At the time, we attributed this discrepancy to the possible interference of external vibrations, expecially a possible vibration of the rigid holder by the detachment or the pulsation of the bubble. Consequently, it was decided to use damped holders. A sponge holder was used.

The early experimental results obtained by using a sponge holder agreed with the theoretical predictions for the damping constant and the frequency. The work proceeded then to assess the effects of polymer additives on the dynamics of bubble pulsation.

The water tank was cleaned upon completion of the tests using polymer additive. A fresh start to measure the damping constant in water was carried out before the next polymer was used. We observed at times that the damping constant scattered widely around the theoretical predictions. The scatter of the results was attributed, at the time, to lack of proper cleaning of the tank, variations in cleanliness of the water, dissolved gas in the water or variations in the cleanliness of the nozzle tip. The frequency of bubble pulsation did not show any noticeable deviation from the predicted one.

At a later stage it was discovered that the length of the protruding part of the nozzle outside the holder in the water significantly affected the damping constant of bubble pulsation. The effect on the pulsation frequency of the bubble was virtually negligible.

We investigated the effects of the length of the protruding part of the nozzle on bubble pulsation, the damping constant and the pulsation frequency. A systematic investigation was conducted using two brass nozzles, 5 and 2.5 mm in diameter respectively. Nozzles of other materials were used. Some of the results appear in later sections.

The results of our investigation are shown in Figures (8.1), (8.1a) and (8.2). Figure (8.1) shows that the effective parameter on the damping constant of bubble pulsation is the protruding ratio rather than the length of the protruding part of the nozzle. The protruding ratio is the ratio of the length of the protruding part of the nozzle to the diameter of the nozzle outlet. Figure (8.1) shows that the damping constant can be as low as 55% of the theoretical value and higher than 125%. For a protruding ratio of 9 and larger the damping constant asymptotes and is no longer affected by any increases in the protruding ratio. At a protruding ratio of 2, approximately, the damping constant acquires a value close to the theoretical value for the free field case. At lower protruding ratios, the damping constant becomes higher. When the protruding ratio approaches zero, the damping constant becomes very high and the frequency of pulsation becomes lower than predicted by the theory of linear pulsation. The dynamics of bubbles in contact with rigid surfaces is a separate subject (Blue (1967)). Still, the analysis of the dynamic behaviour of bubbles in contact with rigid surfaces predicts a frequency of the bubble smaller than that predicted by the linear pulsation theory.

Figure (8.1) shows also that holders of different materials have the same qualitative effects although their quantitative effects are different. Even in the absence of a holder, the nozzle does affect the damping constant though apparently less quantitatively. Some of

the results will be given in the next section. In Figure (8.1a) a schematic diagram of the nozzle and the holder is presented.

In Figure (8.2), we present the results of the effects of the protruding ratio on the frequency of bubble pulsation. As shown in the figure, the effects are very small and lie within experimental error. We are of the opinion that the negligible effects of the protruding ratio on the frequency of bubble pulsation was the reason why previous investigators failed to observe the effects of the protruding ratio on the damping constant and in fact, our failure to discover the effects at an earlier stage of this work.

The mechanism that causes this anomalous behaviour of the damping of bubble pulsation is not clear. However, when the bubble pulsates under the influence of an oscillating pressure field, the damping constant changes accordingly. The damping constant of a forced pulsating bubble depends on the frequency of the pressure field. Prosperetti (1977) investigated the dynamics of bubble pulsation in an oscillating pressure field. His analysis showed that the damping constant of a bubble pulsating in a pressure field retains its free pulsation value only when the frequency of the pressure field is the same as that of the resonance frequency of the bubble. When the two frequencies are not equal, the damping constant of the forced pulsating bubble may become larger or smaller than the free pulsation damping constant. According to Prosperetti (1977), the external pressure field affects both the acoustic and the thermal behaviour of the bubble. The viscous effects of the pulsating bubble do not seem to be affected by the frequency of the pressure field.

No external steady-state pressure fields were applied on the test bubble. It is unlikely that such a field was introduced by any external

source. The experimental values of the frequency of bubble pulsation were in agreement with the theoretical predictions while those of the damping constant varied greatly. Our experimental results cannot be accommodated with the analysis of Prosperetti (1977). A bubble pulsating in a forced oscillating pressure field acquires the frequency of the field rather than its resonance frequency. Our results showed a consistant trend where the pulsating bubble retained its resonance frequency. Therefore, it seems that the existing theoretical analyses of the dynamic behaviour of bubble pulsation, free and forced, do not afford an explanation of the experimental results of Figures (8.1) and (8.2). This in no way means that no external forces are involved in the anomalous behaviour of the bubble. The effects of such forces, in case they exist, have not yet been dealt with analytically.

The results of Figure (8.1) show that the protruding ratio is an effective parameter on the dynamic behaviour of the bubble. The results show also that the stiffness of the holder has a noticeable effect on the bubble behaviour. In the next section, it will be shown that a nozzle floating freely in liquids and held to no holder also affects the dynamic hehaviour of the bubble. These observations would suggest that an interaction between the pulsating bubble, the nozzle and the holder could be responsible for the effects on the bubble pulsation. This interaction could take the form of an acoustic coupling whereby the bubble pulsation would excite the nozzle and the holder. The response of the nozzle and the holder to such an excitation would in turn influence the bubble. In this respect, we would think of two possibilities. The first possibility is that the nozzle vibrates under the influence of the bubble pulsation. The vibration of the nozzle would result in the emission of acoustic waves that may affect the

bubble behaviour. The second possibility is that the sound waves radiated by the bubble are reflected by the steel tank, the water surface, the nozzle or the structure of the holder. The reflected waves may influence the bubble pulsation and cause the anomalous behaviour of the pulsating bubble as shown in Figure (8.1).

However, by examining the results of Figure (8.1), it is shown that the damping constant of the bubble pulsation may be roughly halved at a protruding ratio of over 8. The damping constant of the test bubbles in this work is attributed mainly to the thermal and acoustic losses. The linear damping theory predicts the acoustic damping constant and the thermal damping constant to be equal for the test bubbles. The viscous damping constant is very small and could be neglected especially in low viscosity solvents, eg. water. Consequently, it seems unlikely that the reduction in the damping constant could be attributed to a reduction in the acoustic damping constant only, or the thermal damping constant only. As will be shown in the next section, it is unlikely that the viscous damping constant is involved or affected by the anomalous behaviour of the bubble pulsation. The analysis of Prosperetti (1977) supports such an argument. Therefore, we would argue that both the acoustic and thermal behaviour of the bubble were affected by an acoustic coupling between the pulsating bubble and its surroundings.

The existence of other experimental parameters that may affect the damping of bubble pulsation cannot be ruled out. We would think of the non-sphericity of the bubble as a possible source of influence on the damping of bubble pulsation. For a non-spherical bubble, the viscous losses may be affected due to the possible generation of shear deviatoric stresses. The linear damping theory takes account of the

normal deviatoric stresses only and does not account for the shear stresses. The acoustic behaviour may be influenced as the radiated sound waves may not be the spherical ones only. However, the thermal behaviour is unlikely to be affected since it depends on the change in volume and is independent of the shape. It is possible also that the presence of the nozzle and the holder affect the flow field around the bubble. The linear damping theory assumes the bubble to pulsate in an infinite mass of liquid with a purely symmetrical spherical flow field. The nozzle and the holder are likely to influence the symmetry of such a flow field.

In short, the actual mechanisms causing the anomalous behaviour of the bubble are not clear and further investigations are still required.

8.4 FREE DAMPING OF GAS BUBBLE PULSATION IN VISCOUS NEWTONIAN LIQUIDS

The damping of bubble pulsation in Newtonian liquids of large viscosity has not yet been widely investigated. Houghton (1963) reported that Meyer and Tamm (1939) investigated the effects of viscosity on the damping and frequency of pulsating bubbles in Glycerol (viscosity 1500 cp at 20^{o}C). Their results showed that the viscosity had the effect of reducing the resonance pulsating frequency of the bubble. Free damping of bubble pulsation has been investigated in water only.

The aim of our investigation of the damping of bubble pulsation in liquids of large viscosity was to assess the effects of viscosity on the damping of free bubble pulsation. We meant also to compare the effects of the viscosity of the Newtonian viscous liquids with the effects of the viscosity of the visco-elastic liquids on the damping

of bubble pulsation. We also aimed at making sure whether the viscosity of the liquid maintains its effect on the pulsation of bubbles greatly influenced by the anomalous mechanisms as discussed in the previous section.

To achieve the above aims, we investigated the behaviour of bubble pulsation in two cases. The first was the behaviour of a bubble pulsating under such conditions that the apparent effects of the anomalous mechanism are negligible, and the second case was investigated under conditions that the effects of the anomalous mechanisms are appreciable. We intended to use more than one highly viscous liquid but, unfortunately, we only managed to get reliable results by using light machine oil (measured viscosity 75 cp at $20^{o}C$). Under the conditions of our experiment it was very difficult to get reliable results by using liquids of higher viscosities. In Glyceren, it was virtually impossible to detect the signal of bubble pulsation. However, the viscosity of the light machine oil was comparable with the measured viscosity of a Polyox WSR-301 solution at a concentration of 6000 wppm. This serves our intention of comparing the effects of Newtonian and non-Newtonian visco-elastic liquids of roughly equal viscosities on the damping of bubble pulsation.

Our experimental results are presented in Tables 8.1 and 8.2 below. In Table 8.1, we present the results of the damping of bubble pulsation in water.

Nozzle Material	Bubble Frequency/Hz	Theoretical Damping Constant	Experimental Damping Constant	% Exp./Theo.
Polythene	1220	.0294	.0285	97
Brass	1150	.0290	.0207	71.4

Table 8.1

Experimental results of bubble pulsation in water

Table 8.1 shows that the behaviour of the bubble produced on the polythene nozzle was in agreement with the analysis of the linear pulsation theory. The behaviour produced on the brass nozzle was affected by the nozzle itself as discussed above. It is worth mentioning here that the brass nozzle of Table 8.1 was situated at a downward inclination and as the frequency of the bubble shows, the volume of the bubble was smaller than when the nozzle was kept horiznotally (see Figure (8.2) for comparing the frequencies). The importance of this point will be dealt with in the next section.

In Table 8.2, we present the results of the damping of bubble pulsation in machine light oil.

Nozzle Material	Bubble Frequency/Hz	Theoretical Damping Constant	Experimental Damping Constant	% Exp./Theo.
Polythene	1420	.0370	.0346 - .0366	94 - 99
Brass	1340	.0360	.0275 - .0295	76 - 83

Table 8.2

Experimental results of bubble pulsation in machine light oil

The results of the bubble produced on the polythene nozzle are in good agreement with the theoretical predictions. The results of the bubble produced on the brass nozzle show that the experimental damping constant is less than the theoretical one. This reflects the persistence of the effect of the nozzle on the damping of bubble pulsation. However, by comparing the ratios of the experimental damping constant to the theoretical damping constant in Tables 8.1 and 8.2, it is revealed that a noticeable change happened in the ratio of the bubble produced at the brass nozzle. This change suggests that the effects of the anomalous mechanisms have been reduced in the oil. At the moment, we cannot offer a judgement on whether the viscosity of the liquid alters the effects of the anomalous mechanisms, per se; we are tempted to argue that the increase in the damping constant in oil is due primarily to the viscosity of the oil. Let us assume that the bubble pulsating at 1340 Hz was pulsating in water under the conditions of Table 8.1. Taking the ratio of 71.4% and multiplying it by the theoretical damping constant of 1340 Hz bubble in water, we obtain a would-be experimental damping constant of .0215. The actual increase in the damping constant due to the viscosity of oil is .007, compared with a theoretical increase of .0064 calculated according to Devin (1959).

Our results above concerning the effects of the liquid viscosity on the damping of bubble pulsation are in agreement with the linear pulsation theory.

8.5 THE EFFECTS OF POLYMER ADDITIVES ON THE SIZE AND FREQUENCY OF BUBBLES

The size of a bubble formed by compressing air in a submerged nozzle in a liquid is affected by the properties of the nozzle and the liquid. Among these properties, one can mention the density of the

liquid, its surface tension and viscosity, the diameter of the nozzle and the speed and the pressure of the compressed air. The study of the mechanism of bubble formation is not a prime aim of this work. The results presented below are more of a descriptive nature than analytical, but we will present our view concerning the effects of polymer additives on the bubble's volume and frequency.

The size of a bubble formed at a submerged nozzle is controlled by two forces; the surface tension force and the buoyancy force. The surface tension force is expressed as

$$F_{ST} = 2\pi r S \cos\theta \tag{8.1}$$

while the buoyancy force

$$F_B = \frac{4}{3}\pi R^3(\rho_\ell - \rho_g)g \tag{8.2}$$

where r is the nozzle radius, R is the bubble radius, S the surface tension, θ the angle of contact at the tripple interface and ρ is the density. The subscripts ℓ and g refer to the liquid and the gas respectively.

For a vertically directed nozzle perfectly wetted by the surrounding liquid, the size of the bubble is calculated by equating Equation (8.1) to Equation (8.2). By neglecting the effects of the gas density, there results

$$R = \left[\frac{3}{2} \frac{rS}{\rho_L g} \right]^{1/3} \tag{8.3}$$

In the case of water-air systems at 20°C (Datta et al (1950))

$$\frac{V}{D} = .231 \tag{8.4}$$

where V, the volume of the bubble and D, the diameter of the nozzle. When the nozzle is not vertically orientated, it is predicted that the

volume of the bubble will be smaller than predicted by Equation (8.3). The buoyancy force would tend to drag the bubble upwards across the plane of the orifice and thus the surface tension would be operative only around a portion of the perimeter. In our experimental work, the nozzles were orientated either horizontally or at a downward inclination. Our attempts to produce bubbles on vertically orientated nozzles were abandoned because of the difficulties we encountered. Accordingly, the sizes of the bubbles in our work do not necessarily agree with Equation (8.3). With the exception of the 5 mm brass nozzle situated horizontally, the sizes of the bubbles were less than predicted by Equation (8.3). Our results are in qualitative agreement with the observations of Datta et al (1950).

In measuring the effects of polymer additives on the size of the bubbles, we made sure the changes in the parameters controlling the size of the bubble are confined to the changes induced by the addition of the polymers only. The other parameters were kept the same.

The effects of the polymer additives on the bubble size are shown in Figure (8.3). The effects of the Polyox WSR-301 on the bubble size seems rather complex. The effects of the polyox and Separan additives are prssented in terms of the percentage changes in the bubble size. The nozzles used in our experiments were made of brass and polythene. The diameters of the nozzles were 5 and 4.7 mm respectively. The size of the bubble formed at a horizontally situated brass nozzle was about 110 mm^3 , ie. very close to the predictions of Equation (8.4) irrespective of the fact that the nozzle's orientation is not vertical. When the same nozzle was situated at a downward inclination, the results of Table 8.1 show a reduction in the size of about 14%. The size of the bubble produced on the 4.7 mm diameter polythene nozzle was calculated

to be about 80 mm^3 or 73% of the size of the bubble produced on the horizontal brass nozzle. By taking account of the inclination of the nozzle and the effect of the diameter, the bubble size would still be over 10% smaller than expected for a brass nozzle of equivalent size. This discrepancy is probably due to lack of perfect wetting of the polythene nozzles. In Chapter 4, we discussed the effects of the tube surface wetting on the damping of liquid column oscillation in a flexible PVC tube.

The effects of the Polyox additives on the size of a bubble produced on a brass nozzle show that the additives have a constant effect in reducing the size of the bubble up to a concentration of about 1000 ppm. McComb and Ayyash (1976) reported a reduction in the size of the bubble in Polyox solutions. Thereafter a further reduction in the size of the bubble occurs. The effects of the low concentration solutions are most probably due to a reduction in the surface tension by the Polyox additives. The further decrease could be attributed to the effects of the viscosity. Datta et al (1950) reported that for a nozzle 0.036 - 0.63 cm in diameter, a hundred fold increase in the viscosity caused a diminution in the bubble volume of about 10%. Our results seem to support the observations of Datta et al.

The effects of Polyox additives at low concentrations on the bubble size produced at polythene nozzles is in qualitative agreement with the effects on the horizontal brass nozzle. The further increase in the volume of the bubble is contradictory to the behaviour of the horizontal brass nozzle. It is likely that the wetting of the polythene nozzle has been improved by the high concentrations of the Polyox additives. We estimated above that the effects of the poor wetting are about 10% but in our case we are dealing with a change of about 30%. Such a change is more than to be accounted for by improving the wetting

of the nozzle.

However, by examining the effects of the Separan additives, it is shown that the low concentrations of Separan had virtually no effect on the size of the bubble. This is probably due to the failure of the Separan additives to change the surface tension or the wetting of the nozzle (see Chapter 4). However, at higher concentrations, the bubble size became larger in confirmity with the trend in the Polyox solutions. The percentage change in the bubble volume is larger than to be accounted for by improving the wetting of the nozzle and the expected reduction in the volume due to the increased viscosity.

In the light of the above results, we are tempted to consider that two parameters are affecting the increase in the bubble volume, the orientation of the nozzle and the visco-elasticity of the polymer solutions. We postulate that the two effects may be connected together and act in a combined way. However, nozzles of other materials were not tested in order to check the importance of the tube material along with the apparent importance of the visco-elasticity of the solution combined with the orientation of the nozzle.

In Figure (8.4), we present the results of the effect of the polymer additives on the frequency of bubble pulsation. As shown in the figure, the change in the frequency of the bubbles can be explained in terms of changes in the bubble volume due to the polymer additives rather than the visco-elasticity of the polymer solutions. The results suggest that it is unlikely that the visco-elastic solutions tested induce any changes in the frequency of the test bubbles. In Figure (8.5), we present the effects of the relaxation time of the visco-elastic liquids on the frequency of bubble pulsation of comparable values of the test bubbles. Figure (8.5) shows that a relaxation time in the range 10^{-4} - 10^{-2} sec has no noticeable effects on the frequency

of bubble pulsation.

8.6 THE EFFECT OF THE POLYMER ADDITIVES ON THE RADIATION OF SOUND BY PULSATING BUBBLES

Pulsating bubbles in liquids radiate sound waves. In Chapter 2, we investigated the bubbles as sound radiators in liquids. The acoustic pressure at the bubble surface is equal to the change in the gas pressure associated with the bubble pulsation. The acoustic pressure at the bubble surface is

$$P_{ac} = \frac{\gamma P_o}{V_o} v \qquad (8.5)$$

Equation (8.5) indicates that the acoustic pressure at the bubble surface is directly proportional to the change in the bubble volume. The acoustic pressure decreases linearly with the distance away from the bubble. However, the viscous properties of the liquid attenuate the pressure of the sound waves. The effective viscous parameter in attenuating the sound waves in Newtonian liquids is known as the bulk viscosity. Strasberg (1956) measured the pressure of the sound waves radiated by a pulsating bubble. The measured values were about 65% of the calculated ones. The effects of attenuation seem to have not been taken into consideration.

Absolute measurements of the pressure of acoustic waves were not of a primary interest in this work. The reported results in this section are a by-product of our work to assess the effects of polymer additives on the damping of bubble pulsation. The published data on the effects of visco-elastic liquids are confined to the propagation of ultrasonic acoustic waves in visco-elastic liquids which are not used as drag-reducers in turbulent shear flow. Consequently, we present our data on their own merits.

The propagation of acoustic waves in visco-elastic solutions has been investigated by Pryor (1954), Hunter and Derdul (1967) and Schwartz (1975). The experimental and analytical results of the above investigators seem to show the attenuation of sound waves in the visco-elastic liquids is less than in the pure solvents. The effects of the bulk viscosity are reduced by elastic effects due to the additives. Barker (1973) measured the effects of polymer additives on the radiated noise in a turbulent boundary layer. His results showed that the Polyox additives caused significant reductions in the radiated noise especially at high frequency. At concentrations higher than 100 wppm, Barker observed a reduction in the effects of the additives. Sanders and Sendek (1969) observed a reduction in the noise of the impact of spheres in Polyox solutions.

Our results are shown in Figure (8.6). The complex effects of the polymer additives have some resemblance to their complex effects on the formation of bubbles discussed in the previous section. The pressure of the acoustic waves radiated by a bubble formed at a horizontal brass nozzle were significantly reduced by the Polyox additives. At a concentration higher than 300 wppm, the reduction was concentration-independent. No systematic observations on the effects of the Separan additives are reported. However, at the early stages of our experimental work, we tested the effects of the Separan additives on the damping of bubble pulsation. The bubbles were formed at a horizontal brass nozzle. Our observations indicated a reduction in the pressure of the radiated sound wave (McComb and Ayyash (1976)).

The effects of polymer additives on the pressure of sound waves radiated by bubbles formed at polythene nozzles are also shown in Figure (8.6). The effects of the polymers are apparently not compatible with their previous effects. The Polyox solutions reduced the attenuation

of the sound waves up to a concentration of 1000 wppm. Thereafter, upon increasing the concentration, the effectiveness of the additives declined. At a concentration over 4000 wppm, the attenuation was higher than in the pure solvent (water). The Separan solutions showed no effect up to a concentration of 300 wppm. Thereafter, the attenuation increased.

As it looks, some of our results are compatible with Barker (1973) and Sanders and Sendek (1969) and some are in agreement with those of Pryor (1954). The quantitative effects are contradictory. It seems likely that the specifics of an experimental set-up contribute to the contradictory effects of the polymer additives. The authors who reported decreased attenuation followed techniques of sending a acoustic pulses in the solutions. In a similar technique to measure the velocity of sound (see Chapter 5), we observed the same effects. However, in the case of sound radiation by pulsating bubbles, the pressure of the radiated sound waves is related to the change in the bubble volume associated with the pulsation. No measurements of the change in bubble volume were made. Therefore, it is difficult to assess the effects of the additives on the change in bubble volume and consequently, the changes on the radiated sound waves.

Pryor (1954) attributed the reduction in the effects of the liquid viscosity, on the attenuation of sound, to the elasticity of the visco-elastic solutions. We would suggest that the change in the volume of the pulsating bubble becomes smaller the more viscous the surrounding liquid. The two hypotheses above taken together would probably explain the results of the bubble formed on the polythene nozzle. At low concentrations of Polyox additives, the effects of the solutions viscosity on the change in bubble volume is small while the effects of

The solutions' elasticity are significant and dominant. At higher concentrations the viscous effects are the dominant ones and would counteract the effects of elasticity. The above two hypotheses combined would probably explain the effects of the Separan additives. As will be shown in the next section, the Separan additives failed to relax the viscous effects on the damping of bubble pulsation, and instead showed higher viscous effects (probably extensional viscosity effects). It becomes possible to argue that the relaxation time of the Separan solutions is smaller than the time constant of the bubble pulsation and consequently, no relaxation effects would be observed. It is also possible to argue that the possibility of extensional viscosity effects cancel the relaxation effects of the elasticity in such a way that the net effects are negligible.

The effects of the Separan additives on turbulent drag-reduction and fluid orifice flow were reported to be significantly reduced by the addition of NaCl. (A typical concentration is 2% by weight of NaCl, ie. a half molar solution. (Morgan (1971), Monti (1974).) As shown in Figure (8.6), the addition of 2% by weight of NaCl significantly reduced the effects of the Separan solution on the bubble sound. Similar effects were observed on the bubble size and the associated frequency.

Although the above hypotheses combined seem to offer a descriptive explanation of the sound radiation associated with the bubbles formed on the polythene nozzle, they do not seem to explain the brass nozzle results. We would think of the nozzle material and orientation as being relevant parameters in this field as they seemed to be in the field of bubble production.

8.7 THE EFFECTS OF POLYMER ADDITIVES ON THE DAMPING OF FREE PULSATING BUBBLES

In Chapter 3, we presented an analytical investigation of the effects of the non-Newtonian terms of the visco-elastic solutions on the damping of bubble pulsation. It was revealed that the elasticity of the solutions significantly reduces the viscous effects on the damping of bubble pulsation. The effects of elasticity are larger the larger the relaxation time of the solutions. We present in this section the experimental results of the effects of Polyox WSR-301 and Separan AP-273 solutions on the damping of bubble pulsation.

In Figure (8.7), we present the theoretical damping constant of a 0.3 cm diameter bubble in liquids of different viscosities. Figure (8.7) gives an idea about the would-be theoretical damping constant in Newtonian liquids of equivalent viscosities to the Polyox solutions used in the experimental tests.

In Figure (8.8), the experimental results of the effects of the Polyox additives on the damping of pulsation are shown. The solid line in the figure represents the theoretical damping constant of the test bubble in Newtonian liquids of equivalent viscosities. The results show that the viscosity of the polymer solutions had no noticeable effects on the damping of bubble pulsation. The viscous effects of the Polyox solutions seem to have been totally relaxaed by the elasticity of the solutions themselves. The experimental results agree with the theoretical analysis presented in Chapter 3.

Figure (8.9) shows the effects of the Polyox additives on the damping of a bubble produced at a polythene nozzle. The experimental damping constant in water was about 10% less than the theoretical value. It was intended to configurate the nozzle in such a way that the

experimental damping constnnt would deviate from the theoretical value. We intended to investigate the effects of the Polyox additives on the pulsation damping on a bubble presumably pulsating under the effects of scattered waves. The results in Figure (8.9) shows that the polymer additives did not affect the effects of the scattered waves. They also show that the elasticity of the Polyox solutions relaxed the viscous effects. These results agree with our above results and with the analysis of Chapter 3.

By examining the results in Figure (3.1), it would be shown that the Polyox solutions, especially the concentrated ones, have a relaxation time larger than 10^{-4} sec. This conclusion agrees with the results of Everage and Gordon (1971). The relaxation time of the polymer solutions is molecular weight-dependent. No information was supplied by the manufacturer on the molecular weight of the polymers we used. By depending on the data available in the literature (Arunachalam (1971), Virk (1975)), the molecular weight of the Polyox WSR-301 is about 5×10^6. According to Everage and Gordon, the relaxation time of a 200 wppm solutions of Polyox WSR-301 is 5.56×10^{-3} sec. At such a relaxation time, the Polyox additives would relax the viscous effects of a solution of viscosity orders of magnitude higher than the measured viscosity of the Polyox aolutions. This would make it difficult to confirm or dispute the possibility of the extensional viscosity. However, the irrotational flow field of the pulsating bubble is a likely configuration for the polymer solutions to exhibit extensional viscosity effects. The magnitude of such effects and the conditions under which they would appear are beyond the scope of this work. Oliver (private communication) is of the opinion that a bubble that changes diameter in a visco-elastic liquid is likely to

exhibit extensional viscosity effects. The effects of the extensional viscosity, in case they existed, were relaxed by the elasticity of the solutions.

The effects of the Separan AP-273 additives are shown in Figures (8.10) and (8.11). In Figure (8.10), we present the absolute changes in the magnitude of the damping constant due to the Separan additives. The experimental results of the effects of the Separan additives are compared with the would-be effects of Newtonian solutions of equivalent viscosities. Figure (8.11) shows the above changes in percentage terms. As the results show, the Separan additives increased the viscous effects on the pulsation damping. The damping constant of the bubble pulsation is bigger than would be expected by the viscous effects of equivalent viscous Newtonian solutions. Below, we discuss different hypotheses that are likely to explain the Separan additives' effects.

In a first hypothesis, we argue that the viscosity measurements of the Separan solutions were carried out at a shear rate of 2500 sec^{-1}. Separan solutions are among the shear thinning solutions, therefore it is expected that the viscosity of these solutions would be higher at low shear rates. The calculated strain rate of the bubble pulsation was shown to have a maximum value of 190 sec^{-1}. Consequently, the Separan solution would have a higher viscosity than the measured ones. The reported data on the effects of the shear rate on a Separan solution of 1000 wppm show that the viscosity of this solution at a shear rate of 10 sec^{-1} is 17.5 cp. The viscous effects of a Newtonian liquid of equivalent viscosity would increase the damping constant of the test bubble very much less than the reported effects of the Separan solutions. Consequently, this hypothesis is not supported.

In a second hypothesis, we argue that the relaxation time of the Separan solutions is very much smaller than the time constant of the bubble. Under such conditions, the results of Figure (3.2) show that the elasticity of the solution contributes to increasing the damping of bubble pulsation. Such a hypothesis requires the relaxation time of the Separan solutions to be orders of magnitude smaller than the reported relaxation time of the Separan solutions (Marshall and Metzner (1967), Seyer and Metzner (1969)). The high molecular weight of the Separan additives would suggest that the relaxation time of their solutions is of the order of the bubble time constant. This hypothesis is not supported by the available analytical and experimental evidence.

In a third hypothesis, we argue that the Separan solutions exhibited extensional viscosity effects and that these effects were larger than to be totally relaxed by the elasticity of the solutions. The available data on the molecular weight of the Separan AP-273 suggest a molecular weight comparable with the Polyox WSR-301 (Virk (1975)). The manufacturer's literature (Dow Chemicals) describe the additive to have a high molecular weight. Marshall and Metzner's data (1967) propose a relaxation time of .95 - 6.8 x 10^{-3} sec. Seyer and Marshall (1969) reported similar results.

Figure (8.11) shows that a Separan solution of 1000 wppm concentration increased the damping constant by about 20%. If we assume the solution to have a relaxation time of 10^{-3} sec, the results of Figure (3.1) would suggest that the extensional viscosity of the solution was about 50 poise. Such a result is in agreement with the reported results of Oliver and Bragg (1974). Such a hypothesis seems a likely explanation of the effects of the Separan additives on the damping of bubble pulsation.

The hypothesis that some visco-elastic solutions could exhibit extensional viscosity effects in a bubble pulsation flow field seem to cast doubt on the universal validity of Yang and Lawson's analysis (1974). Their analysis implies that the dynamic viscosity rather than the extensional viscosity is the effective viscous parameter. Our hypothesis and the experimental results of the Separan solutions seem to support the predictions of Ting (1975) and Oliver (private communication) that the extensional viscosity effects are a likely effective parameter in the damping of bubble pulsation in visco-elastic solutions.

8.8 THE EFFECTS OF NaCl ADDITIVES ON THE DAMPING OF BUBBLE PULSATION IN SEPARAN SOLUTIONS

The discovery of the drag-reduction by polymer additives suggested the possibility of using these additives to increase the speed of vessels at sea. Such a practical application of the drag-reducing additives proposed the necessity to assess the effects of the metallic salts on the properties of the drag-reducing solutions.

Morgan (1971) investigated the effects of the NaCl additives (2% by weight) on the orifice flow. His results showed that the NaCl additives significantly reduced the effectiveness of the Separan additives in increasing the pressure drop in an orifice flow. The increase in the pressure drop in the orifice flow is attributed to the extensional viscosity effects of these solutions.

Monti (1974) investigated the effects of NaCl additives (0 - 4% conc.) on the heat transfer properties of the Separan solutions in trubulent pipe shear flow. The NaCl additives were found to reduce the effectiveness of the Separan solutions in reducing the heat transfer.

We carried out an experiment to assess the effects of the NaCl

additives on the damping of bubble pulsation in a 1500 wppm Separan AP-273 solution. The concentration of the NaCl in the solution was 2% by weight as used by both Morgan and Monti.

However, the effects of the NaCl additives on the damping of bubble pulsation in water was investigated up to a concentration of 4% by weight. The results showed no noticeable effects of the NaCl additives in water.

The effects of the NaCl additives on the damping of bubble pulsation in a 1500 wppm solution is shown in Figure (4.10) and (4.11). The NaCl additives relaxed the visco-elastic effects of the solution on the damping of bubble pulsation. Our experimental rssults are in agreement with those of Morgan and Monti. The NaCl additives seem to reduce the extensional viscosity of these solutions.

The NaCl additives resulted in relaxing the effects of the Separan solution on bubble formation. The size of the bubble formed in the half molar Separan solution retained its value in water. The frequency of the pulsating bubble was in agreement with Minnaert's analysis.

In conclusion, the addition of NaCl additives resulted in relaxing the extensional viscosity effects of a 1500 wppm Separan solution on the damping of a bubble pulsation.

CHAPTER 9

CONCLUSION

In the preceding chapters, we presented some aspects of the effects of drag-reducing polymer additives on oscillatory fluid flow. We investigated the effects of the additives on the damping of bubble pulsation and the damping of liquid column oscillation. We also presented some experimental results on the effects of the adsorbed layers of Polyox WSR-301 on the skin friction of turbulent shear flow and liquid column oscillation.

Our experimental results on the adsorbed layers of Polyox showed that these layers yielded inconsistent effects on the turbulent drag of shear flow. In some cases, a reduction in the turbulent drag was observed whereas in other cases no such effects were observed. In fact, this is not entirely surprising in view of the contradictory results reported by other authors as discussed previously.

The adsorbed layers showed a consistent effect in reducing the damping of laminar liquid column oscillation. It was postulated that this effect is associated with improving the wetting of the tube surface, due to the adsorbed layers, rather than inducing changes in the structure of the bulk flow.

In general, we confirmed McComb's observations (1974) that layers of Polyox were adsorbed on the surface of flexible PVC tubes. These layers were of semi-permanent nature, and were not washed off by turbulent shear flow of water. These facts make the layers a favourable subject for further investigation towards revealing their real nature and possible relevance to turbulent drag-reduction.

The effects of the drag-reducing polymers and Aerosol additives on the damping of laminar liquid column oscillation were investigated experimentally. Our results showed that both the Polyox and Aerosol additives reduced the damping of liquid column oscillation in a flexible PVC tube and increased the total number of oscillations of the column. The Polyox additives showed larger quantitative effects on the total number of oscillations. The maximum effects of both additives on the damping constant of liquid column oscillation were virtually equal. No anomalous effects which may be attributed to the elasticity of the Polyox solutions were observed. The Separan additives showed no effects on either the total number of oscillations or the damping constant.

When tested in a glass manometer, all the above additives showed no effects on either the total number of oscillations or the damping constant, at least at low concentrations. At high concentrations a reduction in the total number of oscillations was observed while no effects on the damping constants was induced.

The effects of the Polyox and Aerosol additives on the behaviour of water column in a flexible PVC tube were thought to be associated with improving the wetting of the tube surface, reducing the surface tension or suppressing the toroidal flow regime associated with the reversal of the flow direction. The absence of any noticeable effects of the additives on liquid column oscillation in the glass manometer was attributed to the fact that the water wetted the glass tube surface and consequently, no further improvements were induced by the additives. The absence of any anomalous behaviour of liquid column oscillation was attributed to the fact that the time period of the column oscillation was much higher than the relaxation time of the solutions.

In short, our results on the damping of Newtonian liquid column agreed with the analytical and experimental results of previous investigators (Ury (1962), Biery (1969), McComb (1974)). Our results on the effects of the visco-elastic properties of the polymer solutions on the damping of liquid column oscillation agreed with the analytical and experimental results of Biery (1964). Our results showed, as did Biery's (1964), that the steady-state rheological properties of the visco-elastic solutions can adequately describe the laminar oscillatory flow of these solutions when the relaxation time of the solutions is much smaller than the time period of the liquid column oscillation.

We would think of the forced oscillation of the liquid column as an extension to this work. In a forced oscillating system, it would be possible to attain low time periods of the liquid column comparable with the relaxation time of the visco-elastic solutions. It is postulated that under such conditions, the elastic effects of the solutions would oppose the viscous effects.

The damping of gas bubble pulsation in Newtonian and non-Newtonian visco-elastic solutions was investigated analytically and experimentally. The analytical results in the visco-elastic solutions showed that the elasticity of the solutions oppose the effects of the viscosity of the solution on the damping of bubble pulsation. This conclusion was based on the assumption that the viscosity of the visco-elastic solutions is numerically equal to their shear viscosity. However, the flow field of a pulsating bubble is an irrotational one. In such a flow field, the visco-elastic solutions can exhibit anomalous viscous effects not accounted for by the existing theoretical analysis. These anomalous effects can significantly increase the damping of bubble pulsation.

The experimental results of the damping of bubble pulsation in

water showed the inadequacy of the existing theoretical analysis. Our results on the effects of the protruding ratio of the nozzle on the damping of bubble pulsation revealed that the linear damping theory of bubble pulsation does not account for the above-reported anomalous behaviour of the bubble. We are of the opinion that an acoustic coupling between the bubble pulsation and the near-by objects (nozzle, holder, etc) could be responsible for this behaviour. It seems that a better understanding of the dynamics of bubble pulsation in Newtonian liquids is still needed. We would think that such an understanding may be facilitated by using the Laser-Doppler techniques in studying bubble pulsation. However, the effects of the protruding ratio on the damping constant of bubble pulsation probably shed some light to explain the scattering of the results of Koger and Houghton (1968). These effects probably cast some doubt on the results of McComb and Ayyash (1976).

Our experimental results on the effects of the drag-reducing polymer additives on the damping of bubble pulsation showed that the existing theoretical analysis is not universally applicable. The existing analysis predicts the elastic properties of the solutions to oppose the viscous effects; the latter supposed to be shear viscosity-dependent. The results of the Polyox WSR-301 additives supported the analysis whereas the results of the Separan AP-273 additives showed its inadequacy. The apparent inadequacy does in no way suggest the invalidity of the hypothesis that the elasticity of the solutions oppose their viscous effects. It simply suggests that the existing analysis does not account for the possibility of anomalous viscous effects of the visco-elastic solutions in irrotational flow fields.

To sum up, we say that drag-reducing polymer additives in laminar oscillatory flow fields can exhibit anomalous behaviour. This behaviour

is likely to be exhibited if the time period of the oscillatory flow is comparable with the relaxation time of the solutions. The effects of the polymer additives on oscillatory fluid flow are being considered as a promising field of research which may contribute to the understanding of the phenomenon of drag-reduction and ultimately the utilizing of its potential engineering applications.

```
C                    APPENDIX (A)
C
C
C
C
C     THIS FORTRAN IV PROGRAM PERFORMS THE CALCULATION
C     OF THE DAMPING CONSTANT OF BUBBLE PULSATIONS IN
C     VISCOELASTIC LIQUIDS
C
C     AN: DAMPING CONSTANT OF NEWTONIAN SOLUTIONS
C     A1: DAMPING CONSTANT OF VISCOELASTIC SOLUTIONS
C
C     T.S.P.L (1)
C     ***********
      DIMENSION RO(100),O(100),X1(100),X2(100),BB(100)
      DIMENSION B1(100),B2(100),B3(100),Y(100),AS(100)
      DIMENSION W(100),BC(100),H(100),V(100),AC(100)
      DIMENSION F(100),HH(100),A0(100),A1(100),AN(100)
      DIMENSION ACO(100),TH(100)
      SO=145000.0
      G=1.4
      PO=1000000.0
      E=1
      READ,M
      DO 20 I=10,M,10
   20 X1(I)=.0001*I
      READ,K
      DO 30 IJ=10,K,10
   30 X2(IJ)=0.00000*IJ
      DEN=1.0
      READ,N
      DO 60 IJ=10,K,10
      WRITE(6,55) (X2(IJ))
   55 FORMAT(4X,'X2= ',F8.6)
      DO 50 J=10,M,10
      WRITE(6,77) (X1(J))
   77 FORMAT(4X,'X1= ',F8.6)
      WRITE(6,66)
   66 FORMAT(2X,'**********************************
     1********')
      DO 10 I=10,N,10
      RO(I)=.0003*I
      O(I)=(1/RO(I))*SQRT(3*G*PO)
      ACO(I)=O(I)**2*RO(I)/SO
      TH(I)=6.28*.000441*(O(I)/6.28)**1.5
      B1(I)=(1+(ACO(I)+TH(I))*X1(J)+4*E*X2(IJ)/
     1(DEN*RO(I)**2))/X1(J)
      B2(I)=(ACO(I)+TH(I)+4*E/(DEN*RO(I)**2)+
     20(I)**2*X1(J))/X1(J)
      B3(I)=O(I)**2/X1(J)
C
C     LISTING CONTINUED ON NEXT PAGE
C
```

```
C
C      APPENDIX (A)  CONTINUED
C
       BB(I)=(2*(B1(I)**3)-9*B1(I)*B2(I)+27*B3(I))/27
       AS(I)=(3*B2(I)-B1(I)**2)/9
       Y(I)=(BB(I)**2/4)+(AS(I)**3)
       W(I)=(-(BB(I)/2)-(Y(I)**.5))
       IF(W(I)-0)2,1,1
     1 BC(I)=W(I)**.3333
       GO TO 5
     2 H(I)=-W(I)
       BC(I)=-H(I)**.3333
     5 CONTINUE
       V(I)=-(BB(I)/2)+(Y(I)**.5)
       IF(V(I)-0)4,3,3
     3 AC(I)=V(I)**.3333
       GO TO 7
     4 HH(I)=-V(I)
       AC(I)=-HH(I)**.3333
     7 CONTINUE
       F(I)=.865*(AC(I)-BC(I))
       A1(I)=((B1(I)/3)+.5*(AC(I)+BC(I)))/(D(I)/2)
       A0(I)=((B1(I)/3)-(AC(I)+BC(I)))/(D(I)/2)
       AN(I)=((4*E/(DEN*RD(I)**2))+ACD(I)+TH(I))/D(I)
    10 CONTINUE
       WRITE(6,99)
    99 FORMAT(4X,'FREQ',10X,'A1',10X,'A0',10X,'AN')
       WRITE(6,88)(F(I),A1(I),A0(I),AN(I),I=10,N,10)
    88 FORMAT(4X,F10.1,2X,F8.5,2X,F8.5,2X,F8.5)
    50 CONTINUE
       WRITE(6,44)
    44 FORMAT(4X,':::::::::::::::::::::::::::::::
      1:::::::::::::')
       WRITE(6,33)
    33 FORMAT(4X':::::::::::::::::::::::::::::::
      2:::::::::::::')
    60 CONTINUE
       END
C
C
C
     C      END OF PROGRAM
C      **************
```

```
C                    APPENDIX (B)
C
C
C
C
C     THIS FORTRAN IV PROGRAM PERFORMS THE CALCULATION
C     OF THE MEAN VALUE OF THE BACKGROUND NOISE
C
C     T.S.P.L (5)
C     ***********
      INTEGER*2 A(500)
      READ(8)A
      B=0
      E=0
      DO 10 J=1,500
      B=B+A(J)
   10 CONTINUE
      C=B/500
      DO 20 J=1,500
      D=(C-A(J))**2
      E=E+D
   20 CONTINUE
      F=SQRT(E/500)
      H=F/C
      PRINT,C,F,H
      END
C
C
C
C     END OF PROGRAM
C     **************
```

```
C                        APPENDIX (C)
C
C
C
C
C     THIS FORTRAN IV PROGRAM PERFORMS THE CALCULATION
C     OF THE PEAKS OF OSCILLATIONS IN A BUBBLE PULSATION
C     SIGNAL
C
C     T.S.P.L (3)
C     ***********
      INTEGER*2 A(500)
      READ(8)A
      J=1
      IB=0
      IC=0
      E=0
      F=0
      N=0
      M=0
      IH=0
      IG=0
      B=A(1)
      C=A(2)
      D=A(3)
      I=3
      IF((C-B).LE.0.AND.(C-D).LT.0) GO TO 10
      IF((C-B).GE.0.AND.(C-D).GT.0) GO TO 11
    2 IF(I-1)21,22,23
   21 STOP
   22 J=J+1
   23 CONTINUE
      I=I+1
      IF(I.EQ.501) GO TO 3
      B=C
      C=D
      D=A(I)
      IF((C-B).LE.0.AND.(C-D).LT.0) GO TO 10
      IF((C-B).GE.0.AND.(C-D).GT.0) GO TO 11
      GO TO 2
    3 B=C
      C=D
      READ(8)A
      I=0
      I=I+1
      D=A(I)
      IF((C-B).LE.0.AND.(C-D).LT.0) GO TO 10
      IF((C-B).GE.0.AND.(C-D).GT.0) GO TO 11
C
C     CONTINUED ON NEXT PAGE
C
```

```
      C
C       APPENDIX (C)  CONTINUED
C
        GO TO 2
   10   CONTINUE
        IB=IC+1
        IH=C
        N=I-1
        IF(IB-50)17,18,18
   18   F=500000./E
        PRINT,F
        STOP
   17   GO TO 2
   11   CONTINUE
        IC=IB+1
        IG=C
        M=I-1
        IF(M-N)12,9,14
   12   M=M+500
        K=M-N
        IF(IC.GE.6) GO TO 5
        GO TO 15
   14   K=M-N
        IF(IC.GE.10) GO TO 5
   15   IF(K-15)9,4,4
    4   IF(IABS(IH)+IABS(IG)-600) 9,5,5
    9   IB=0
        IC=0
        N=0
        M=0
        GO TO 2
    5   E=E+K
        WRITE(1,200)IB,IH,N
  200   FORMAT('IB',I6,2X,'IH',I6,2X,'N',I6,)
        WRITE(1,201)IC,IG,M,K,J
  201   FORMAT('IC',I6,2X,'IG',I6,2X,'M',I6,2X,
       1'K',I6,2X,'J',I6,)
        GO TO 2
        END
C
C
C
C       END OF PROGRAM
C       ***************
```

```
C                    APPENDIX (D)
C
C
C
C
C     THIS FORTRAN IV PROGRAM PERFORMS THE CALCULATION
C     OF THE PEAKS OF THE AVERAGE BUBBLE PULSATION SIGNAL
C
C     T.S.P.L (2)      ,
C     ***********
      DIMENSION IW(25),IX(25),IY(25),IZ(25)
      DIMENSION A(25),B(25),C(25),D(25),E(25),F(25)
      DIMENSION IB(250),IH(250),IC(250),IG(250)
      DIMENSION IM(25),IN(25),IJ(25),IK(25)
      DO 10 I=1,250
      READ(4,11)IB(I),IH(I),N
   11 FORMAT(2X,I6,4X,I6,3X,I6)
      READ(4,12)IC(I),IG(I),M
   12 FORMAT(2X,I6,4X,I6,3X,I6)
   10 CONTINUE
      DO 30 J=1,25
      IM(J)=0
      IN(J)=0
      IK(J)=0
      IJ(J)=0
   30 CONTINUE
      DO 20 J=1,25
      DO 20 I=J,250,25
      IM(J)=IM(J)+IB(I)
      IN(J)=IN(J)+IH(I)
      IJ(J)=IJ(J)+IC(I)
      IK(J)=IK(J)+IG(I)
   20 CONTINUE
      DO 40 J=1,25
      IW(J)=(IM(J)/10)
      IX(J)=(IN(J)/10)-30
      IY(J)=(IJ(J)/10)
      IZ(J)=(IK(J)/10)-30
C
C     CONTINUED ON NEXT PAGE
C
```

```
C
C      APPENDIX (D)  CONTINUED
C
   40 CONTINUE
      DO 70 J=1,25
      D(J)=0
      F(J)=0
   70 CONTINUE
      DO 50 J=1,25
      DO 50 I=J,250,25
      D(J)=D(J)+(IX(J)-IH(I))**2
      F(J)=F(J)+(IZ(J)-IG(J))**2
   50 CONTINUE
      DO 60 J=1,25
      A(J)=SQRT(D(J)/10)
      B(J)=A(J)/IX(J)
      C(J)=SQRT(F(J)/10)
      E(J)=C(J)/IZ(J)
   60 CONTINUE
      WRITE(8,200)(IW(J),IX(J),B(J),IY(J),IZ(J)
     1,E(J),J=1,25)
  200 FORMAT(2I6,2X,F5.3,2X,2I6,2X,F5.3)
      END
C
C
C
C      END OF PROGRAM
C      **************
```

```
C                        APPENDIX (E)
C
C
C
C
C     THIS FORTRAN IV PROGRAM PERFORMS THE NORMALIZATION
C     OF THE PEAKS OF THE AVERAGE BUBBLE PULSATION SIGNAL
C
C     T.S.P.L (4)
C     ***********
      DIMENSION IX(25),A(25)
      DIMENSION IY(25),C(25)
      DO 10 I=1,25
      READ(9,11)IX(I),IY(I)
   11 FORMAT(6X,I6,15X,I6)
   10 CONTINUE
      DO 20 I=1,25
      IF(IX(I+1)-IX(I)) 20,20,19
   20 CONTINUE
   19 B=IX(I)
      DO 50 I=1,25
      IF(IY(I+1)-IY(I)) 51,50,50
   50 CONTINUE
   51 D=IY(I)
      DO 30 I=1,25
   30 A(I)=IX(I)/B
      DO 60 I=1,25
   60 C(I)=IY(I)/D
      WRITE(6,101)(I,A(I),I,C(I),I=1,25)
  101 FORMAT(2X,I3,2X,F5.3,4X,I3,2X,F5.3)
      END
C
C
C
C     END OF PROGRAM
C     **************
```

```
C                        APPENDIX (F)
C
C
C
C
C     THIS FORTRAN IV PROGRAM PERFORMS THE CALCULATION
C     OF THE LEAST SQUARE LINE FOR BEST-FIT OF THE
C     DAMPING CONSTANT, DATA OF THE BUBBLE PULSATION
C     SIGNAL
C
C     T.S.P.L (6)
C     ***********
      DIMENSION X(20),Y(20),S(20),Z(20),W(20)
      G=0
      E=0
      C=0
      B=0
      D=0
      READ,F
      READ,N
      DO 50 I=1,N
   50 READ,Y(I),X(I)
      DO 10 I=1,N
   10 S(I)=ALOG(Y(I))
      DO 20 I=1,N
      B=B+X(I)
      C=C+S(I)
      D=D+(X(I)*S(I))
      E=E+(X(I))**2
   20 CONTINUE
      A2=((C*B)-(D*N))/(B**2-(E*N))
      A1=(C-(B*A2))/N
      WRITE(6,66)A1,A2
   66 FORMAT(2X,'A1',2X,F6.3,2X,'A2',2X,F6.4)
      DS=.014
      DTH=.000441*SQRT(F)
      DT=DS+DTH
      H=-A2/3.14
      PER=H/DT
      WRITE(6,55)DT,H,PER
   55 FORMAT(2X,'DT',2X,F6.4,2X,'H',2X,F6.4,2X,'PER'
     1,2X,F6.4)
      V=EXP(A1)
      DO 30 I=1,N
      W(I)=EXP(A2*X(I))
      Z(I)=V*W(I)
   30 CONTINUE
      WRITE(6,101)(Z(I),W(I),I=1,N)
  101 FORMAT(2X,F6.3,2X,F6.3)
      DO 70 I=1,N
   70 G=G+((Y(I)-W(I))/W(I))**2
      P=SQRT(G)
      R=P/N
      PRINT,R
      END
C
C
C
C     END OF PROGRAM
```

```
C                          APPENDIX (G)
C
C
C
C
C     THIS FORTRAN II PROGRAM PERFORMS ALL THE
C     PROCESSES INCORPORATED IN APPENDICIES A TO F.
C
C
C     T.S.P.L 7
C     *********
C
C     TWO CHANNEL LOG ROUTINE
C     ***********************
      COMMON IRE(6000),IPEAK(50,10),IFREQ(50,10),DEV(50)
     1,S(50),AMEAN(50)
      LA=&4
      IEXPT=0
    6 CALL PLOT(-1,0,0)
      CALL PLOT(2,-20000,5000)
      WRITE(1,128)
  128 FORMAT('    WAITING FOR DATA    ')
      NPT=6000
      MAX=0
      DC=0
      IREM=1
      IEXPT=IEXPT+1
      L=1
    2 CALL RDCT(IA,2,8,5)
      IF(IBYTE(IA,0)-168)2,2,3
    3 CALL RDCT(IRE,NPT,8,5)
      CALL PLOT(-1,0,0)
      DO 10 I=1,NPT
      CALL IBYTE(IRE(I),0,JJ)
      DC=DC+FLOAT(JJ)
   10 IRE=JJ-128
      DC=DC/6000-128
      WRITE(1,105)DC
  105 FORMAT(' DC'=,F10.5)
      IX=32000
      IDX=20
      CALL PLOT(0,-32000,0)
      DO 1 I=2,5999
      IRV=IABS(IRE(I+1)+IRE(I)*2+IRE(I-1))
      IF(IRV-MAX)8,7,7
    8 IF(I-IREM-20)15,15,16
   16 IPEAK(L,IEXPT)=100*IRE(IREM)-IFIX(100*DC)
      IFREQ(L,IEXPT)=IREM
      IX1=FLOAT(IREM)*FLOAT(IDX)-32000
      CALL PLOT(0,IX1,0)
      CALL PLOT(1,IX1,IY)
      CALL PLOT(0,IX,IY)
C
C     CONTINUED ON NEXT PAGE
C
```

```
      C
C       APPENDIX (G) CONTINUED
C
        L=L+1
        IREM=I
        MAX=0
        IF(L-50)7,7,14
      7 IREM=I
        MAX=IAV
     15 IF(IABS(IX)-32000)4,4,5
      5 IX=-32000-IDX
        CALL PLOT(0,IX,IY-20000)
      4 IX=IX+IDX
        IY=IRE(I)*256
      1 CALL PLOT(1,IX,IY)
     14 CALL PLOT(-1,0,0)
        DO 9 I=1,49
        IFREQ(I,IEXPT)=-IFREQ(I,IEXPT)+IFREQ(I+1,IEXPT)
      9 WRITE(1,100)IPEAK(I,IEXPT),IFREQ(I,IEXPT)
     21 WRITE(1,103)
    103 FORMAT('DO YOU WANT TO KEEP THE LAST RUN(Y OR N)')
        CALL IPCH(1,I)
        CALL MATCH(I,'Y','N',J)
        GOTO(21,22,23),J
     23 IEXPT=IEXPT-1
     22 WRITE(1,101)IEXPT
    101 FORMAT('ANOTHER EXPERIMENT?(Y OR N)-',I2,
       1' SAMPLES TAKEN SO FAR')
        CALL IPCH(1,J)
        CALL MATCH(J,'Y','N',I)
        GO TO (6,6,11),I
    100 FORMAT(2I8)
C       PROCESSING IPEAK
C       ****************
    11  AMAX=0
        DO 20 I=1,50
        AMEAN(I)=0
        DEV(I)=0
        DO 19 J=1,IEXPT
     19 AMEAN(I)=AMEAN(I)+FLOAT(IPEAK(I,J))
        AMEAN(I)=AMEAN(I)/FLOAT(IEXPT)
        IF(ABS(AMEAN(I))-AMAX)51,51,52
     52 AMAX=ABS(AMEAN(I))
     51 DO 18 J=1,IEXPT
     18 DEV(I)=DEV(I)+(AMEAN(I)-FLOAT(IPEAK(I,J))**"
        DEV(I)=SQRT(DEV(I)/FLOAT(IEXPT)/AMEAN(I)
     20 WRITE(1,102)AMEAN(I),DEV(I)
    102 FORMAT(2F14.5)
C
C       CONTINUED ON NEXT PAGE
C
```

```
C
C      APPENDIX (G) CONTINUED
C
C      DAMPING CONSTANT
C      ****************
       CALL PLOT(-1.0,0)
       F=0
       DO 53 I=2,49
    53 F=F+FLOAT(IFREQ(I,1))
       F=3840000/F
       WRITE(1,123)F
   123 FORMAT( 'F=' ,F10.5)
       AN=14
       WRITE(1,122)AMAX
   122 FORMAT('AMAX=' ,F12.5)
       L=17
       M=43
       AMULT=1
       DO 54 K=1,2
       B=0
       C=0
       D=0
       E=0
       DO 50 I=L,M,2
       S(I)=ALOG(AMULT*AMEAN(I)/AMAX)
       X=FLOAT(I-L)/2
       B=B+X
       C=C+S(I)
       D=D+S(I)*X
    50 E=E+X**2
       L=18
       M=44
       AMULT=-1
       GO TO (56,57),K
    56 WRITE(1,124)
       GO TO 58
    57 WRITE(1,125)
   124 FORMAT('          *****POSITIVE PEAKS*****')
   125 FORMAT('          *****NEGATIVE PEAKS*****')
    58 A2=(C*B-D*AN)/(B**-E*AN)
       A1=(C-B*A2)/AN
       WRITE(1,120)A1,A2
   120 FORMAT('      A1= ',F10.5,//      A2= ',F10.5)
       DS=.014
       DTH=.000441*SQRT(F)
       DT=DS+DTH
       H=-A2/3.14159
       PER=H/DT
       WRITE(1,121)DT,H,PER
   121 FORMAT('  DT = ',F9.5,//  H=' ,F9.5,//
      1PER=',F9.5,'%')
    54 CONTINUE
       STOP
       END
C
```

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Head of the Department,
Professor J. T. Davies

Professor S. R. M. Ellis
Professor P. J. Garner

Ref: DRO/VCI

4th November, 1976

Mr. S. Ayyash,
Mechanical Engineering Department,
Edinburgh University,
Edinburgh.

Dear Mr. Ayyash,

I enclose the reprint you asked for, together with several others.

Blowing a bubble in liquid, or inflating a spherical shell of liquid both provide a system in which a rather ideal form of biaxial extensional flow exists. I once tried blowing bubbles in Lyle's Golden syrup (μ_{SHEAR} about 500 poise at 20°C) in order to measure the extensional viscosity. The results were of the right order of magnitude, but the arrangement was not easy to use, and worked only for very viscous liquids, if they were Newtonian. I tried the same thing with 2.5% Polyox solutions and got some most odd results!

So the answer to your question is basically "yes" - extensional viscosity effects should appear in the flow field around a bubble or shell which is changing diameter.

Yours sincerely,

Dr. D.R. Oliver

Encl.

FIGURE CAPTIONS

Figure 3.1 Effect of the relaxation time on the damping constant of bubble pulsation in liquids of various viscosities.

Figure 3.2 Effect of the relaxation time on the damping constant of bubble pulsation.

Figure 3.3 Effect of the retardation time on the damping constant of bubble pulsation.

Figure 4.1 The experimental set-up.

Figure 4.2 Manometer with characteristic dimensions.

Figure 4.3 The preliminary test indicating drag-reduction in a well-aged coated tube. □ clean tube. ○ tube with coating of Polyox WSR-301.

Figure 4.4 A representative result indicating no drag-reduction in a freshly-prepared coated tube. □ clean tube. ○ tube with coating of Polyox WSR-301.

Figure 4.5 Effect of wall coating on manometer oscillations both before and after shear flow. □ clean tube. X coated tube before shear flow test. ○ coated tube after shear flow test.

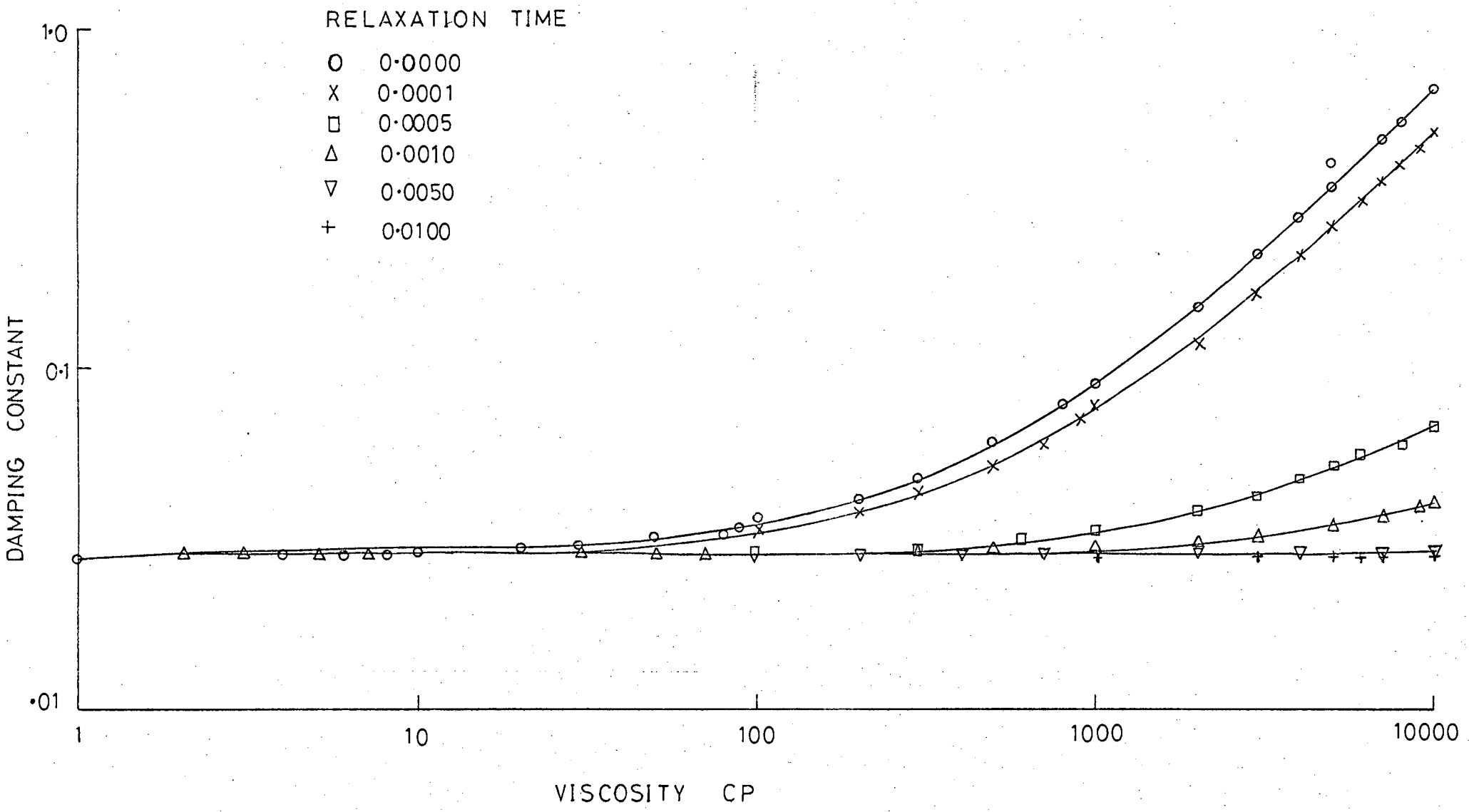
Figure 4.6 Damping of manometer oscillations in flexible PVC tube.

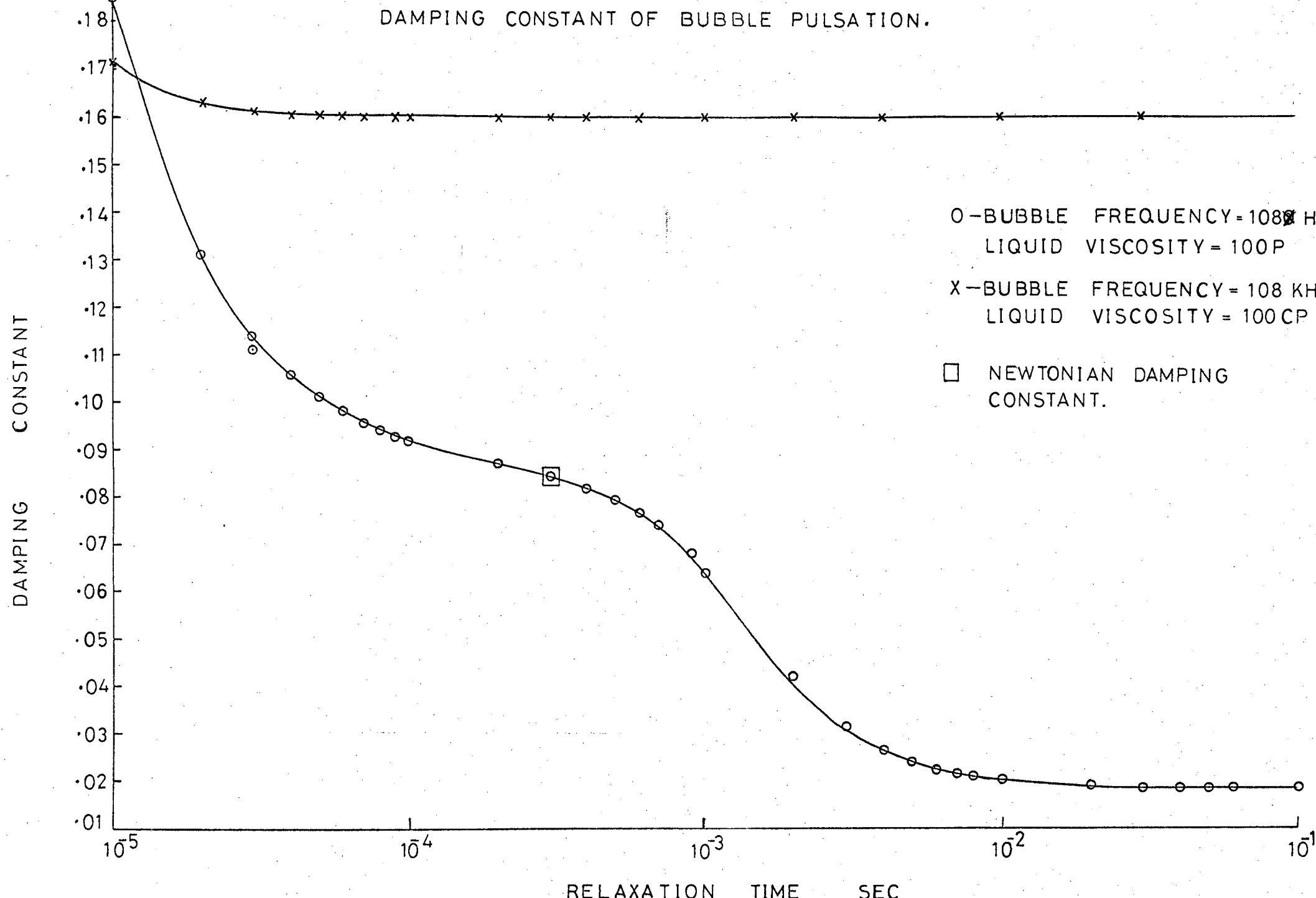
Figure 4.7 Decay of oscillation amplitude of a liquid column.

Figure 4.8 Experimental results of the damping constant of liquid column oscillation.

Figure 4.9 Effect of Polyox WSR-301 on the damping of liquid column oscillation in a flexible PVC manometer.

FIG. 3.1 EFFECT OF THE RELAXATION TIME ON THE DAMPING CONSTANT OF BUBBLE PULSATION IN LIQUIDS OF VARIOUS VISCOSITIES.




DAMPING CONSTANT OF BUBBLE PULSATION.
DAMPING CONSTANT
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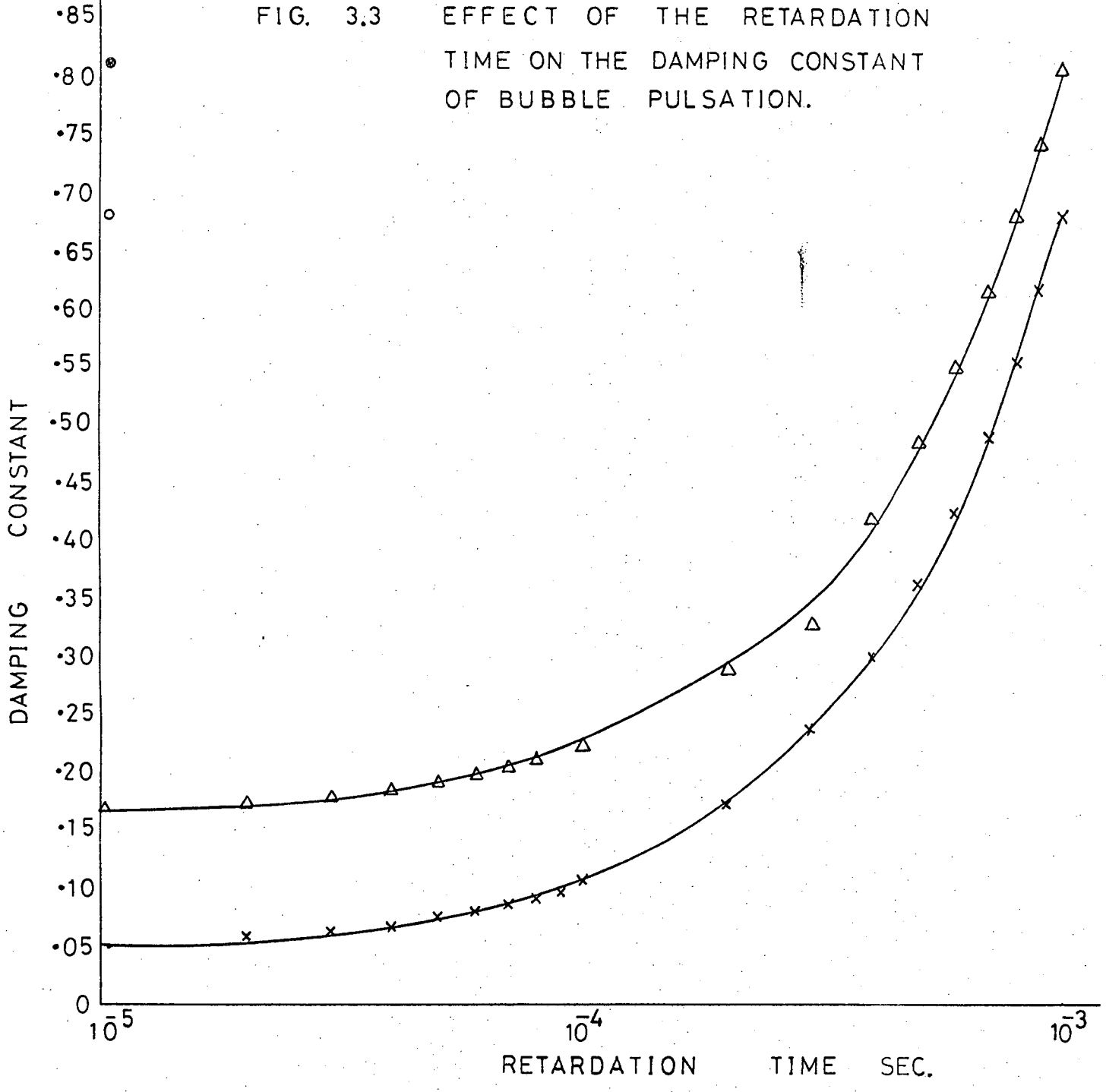
·03
·02
·01
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
RELAXATION TIME SEC
O—BUBBLE FREQUENCY = 108 KH
LIQUID VISCOSITY = 100 P
X—BUBBLE FREQUENCY = 108 KH
LIQUID VISCOSITY = 100 CP
NEWTONIAN DAMPING CONSTANT.

FIG. 3.3 EFFECT OF THE RETARDATION TIME ON THE DAMPING CONSTANT OF BUBBLE PULSATION.



X BUBBLE FREQUENCY 1080HZ
VISCOSITY = 100 P
RELAXATION TIME = 10^{-3} SE
oNEWTONIAN DAMPING CONSTA

Δ BUBBLE FREQUENCY 108KHZ
VISCOSITY = 100 CP
RELAXATION TIME = 10^{-3} SE
●NEWTONIAN DAMPING CONST

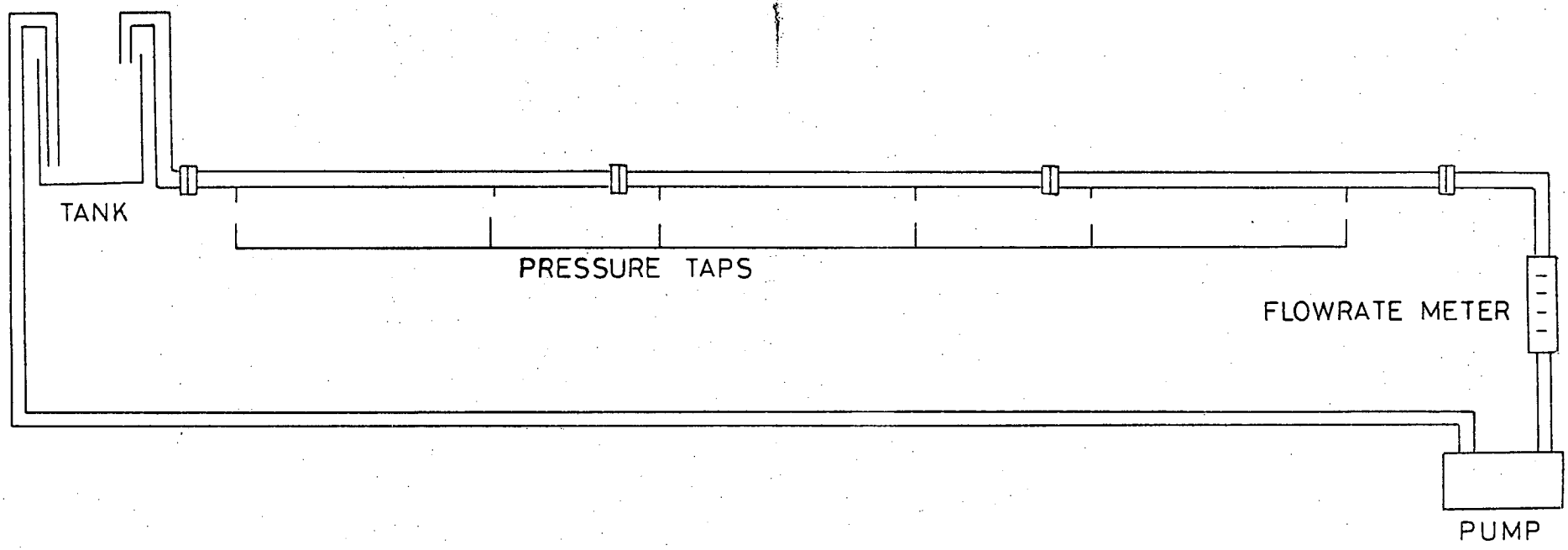


FIG. 4.1 THE EXPERIMENTAL SET-UP.

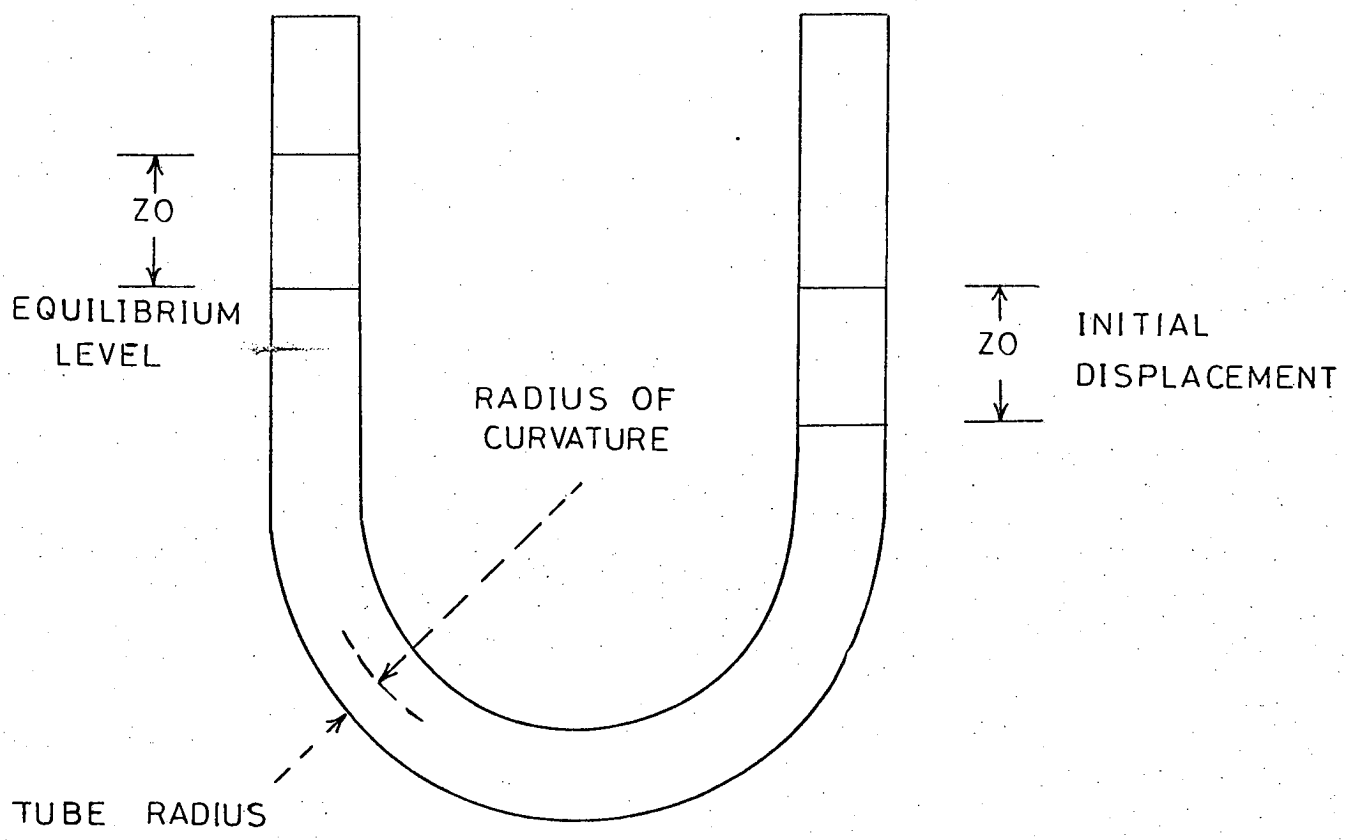


FIG. 4.2 MANOMETER WITH CHARACTERISTIC DIMENSIONS.

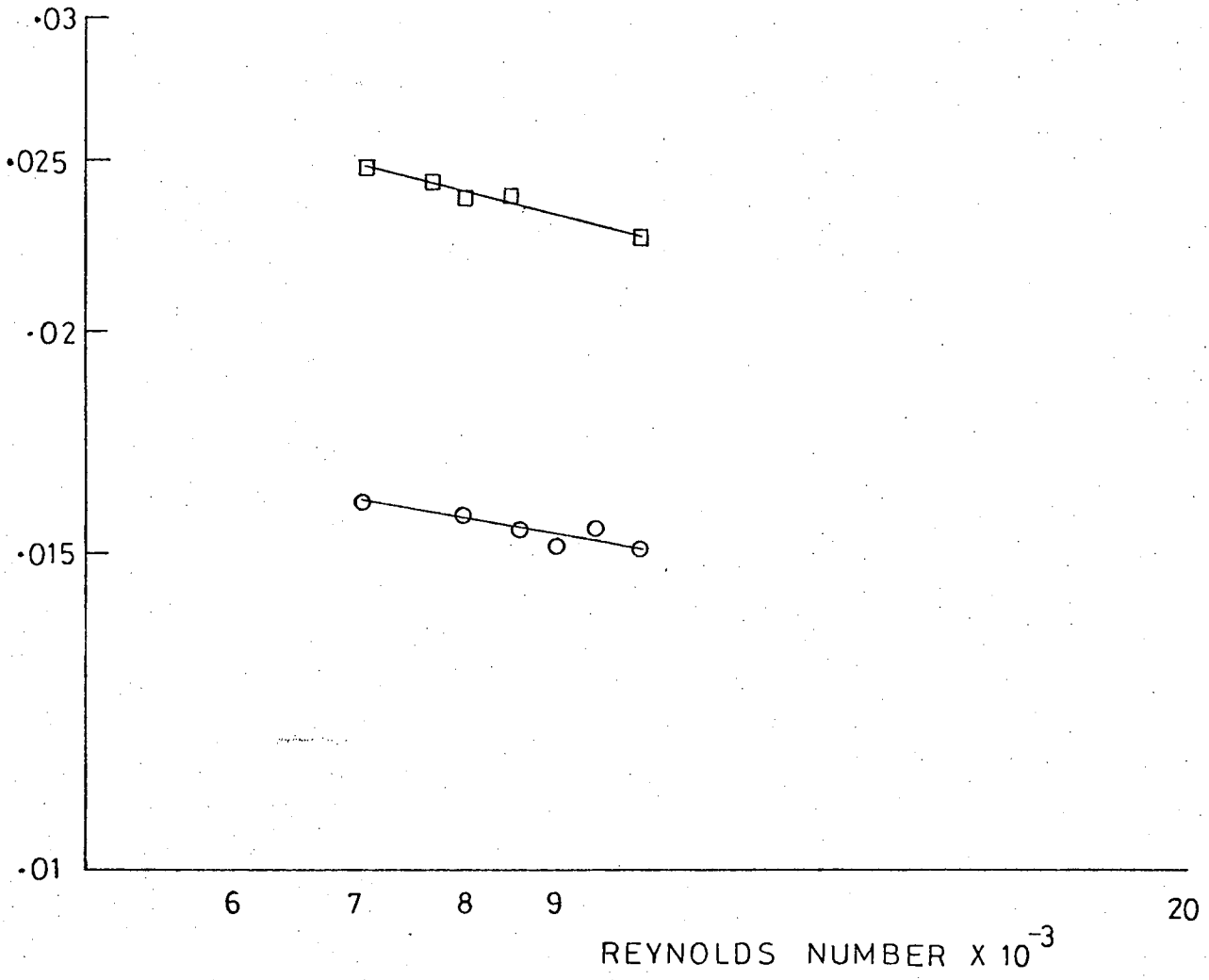


FIG 4.3

The preliminary test indicating drag-reduction in a well-aged coated tube. □ clean tube. ○ tube with coating of Polyox WSR-301.

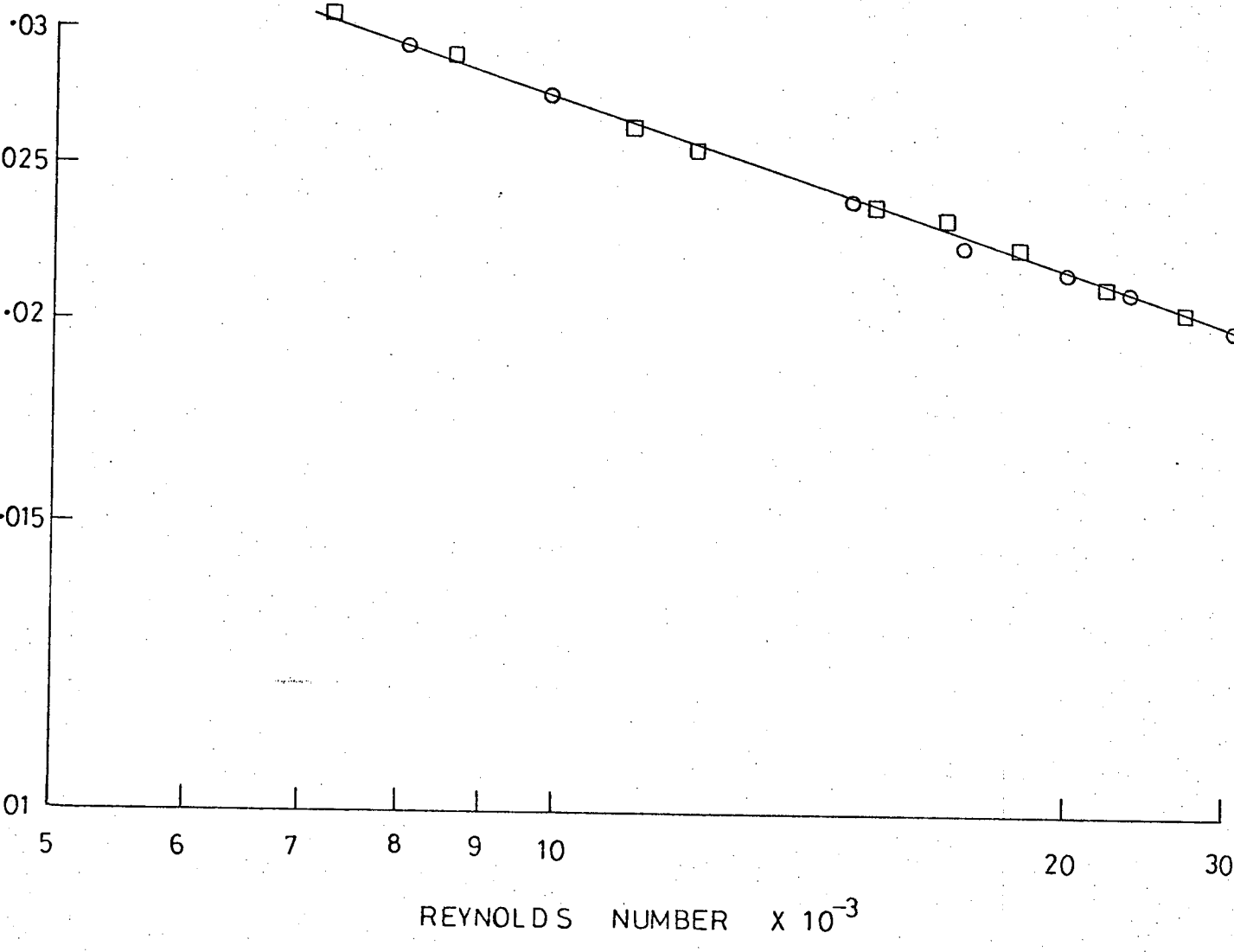


FIG. 4.4

A representative result indicating no drag-reduction in a freshly-prepared coated tube. □ clean tube. O tube with coating of Polyox WSR-301.

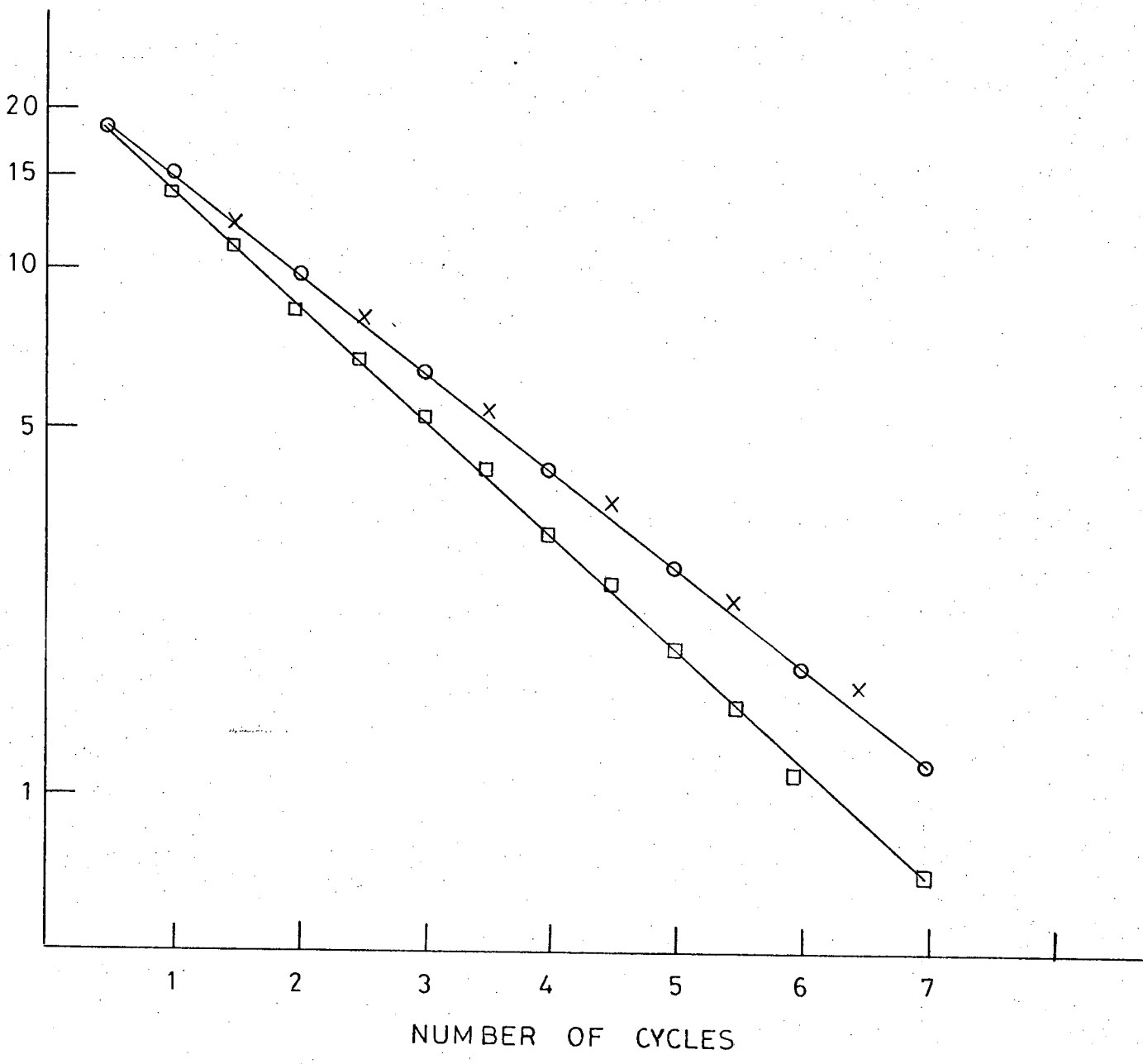


FIG 4.5

Effect of wall coating on manometer oscillations both before and after shear flow. □ clean tube. × coated tube before shear flow test. ○ coated tube after shear flow test.

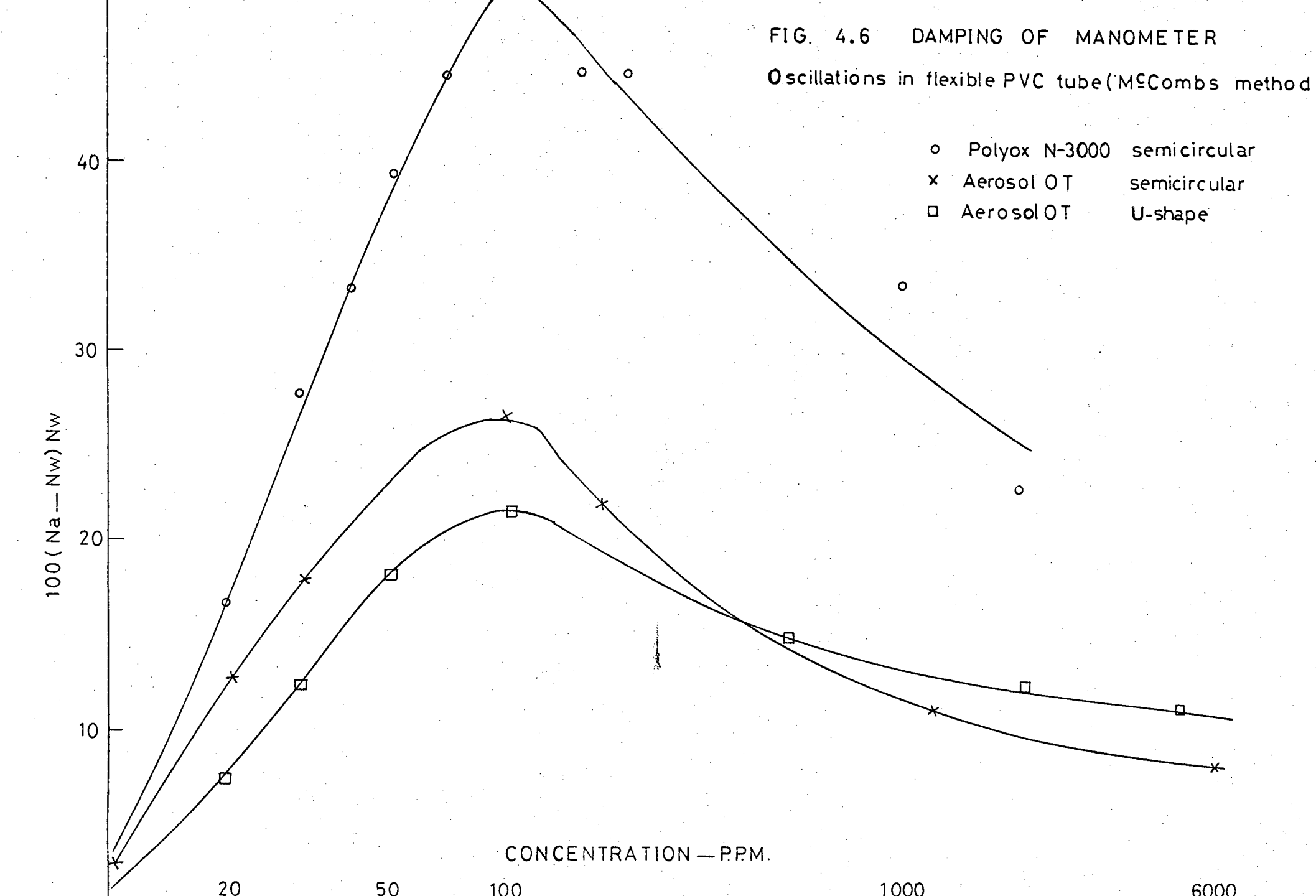


FIG. 4.6 DAMPING OF MANOMETER
Oscillations in flexible PVC tube (McCombs method
o Polyox N-3000 semicircular
× Aerosol OT semicircular
□ Aerosol OT U-shape
100 (Na — Nw) Nw
40
30
20
10
CONCENTRATION — P.P.M.
20
50
100
1000
6000

FIG 4.7 DECAY OF OSCILLATION AMPLITUDE OF A LIQUID COLUMN.

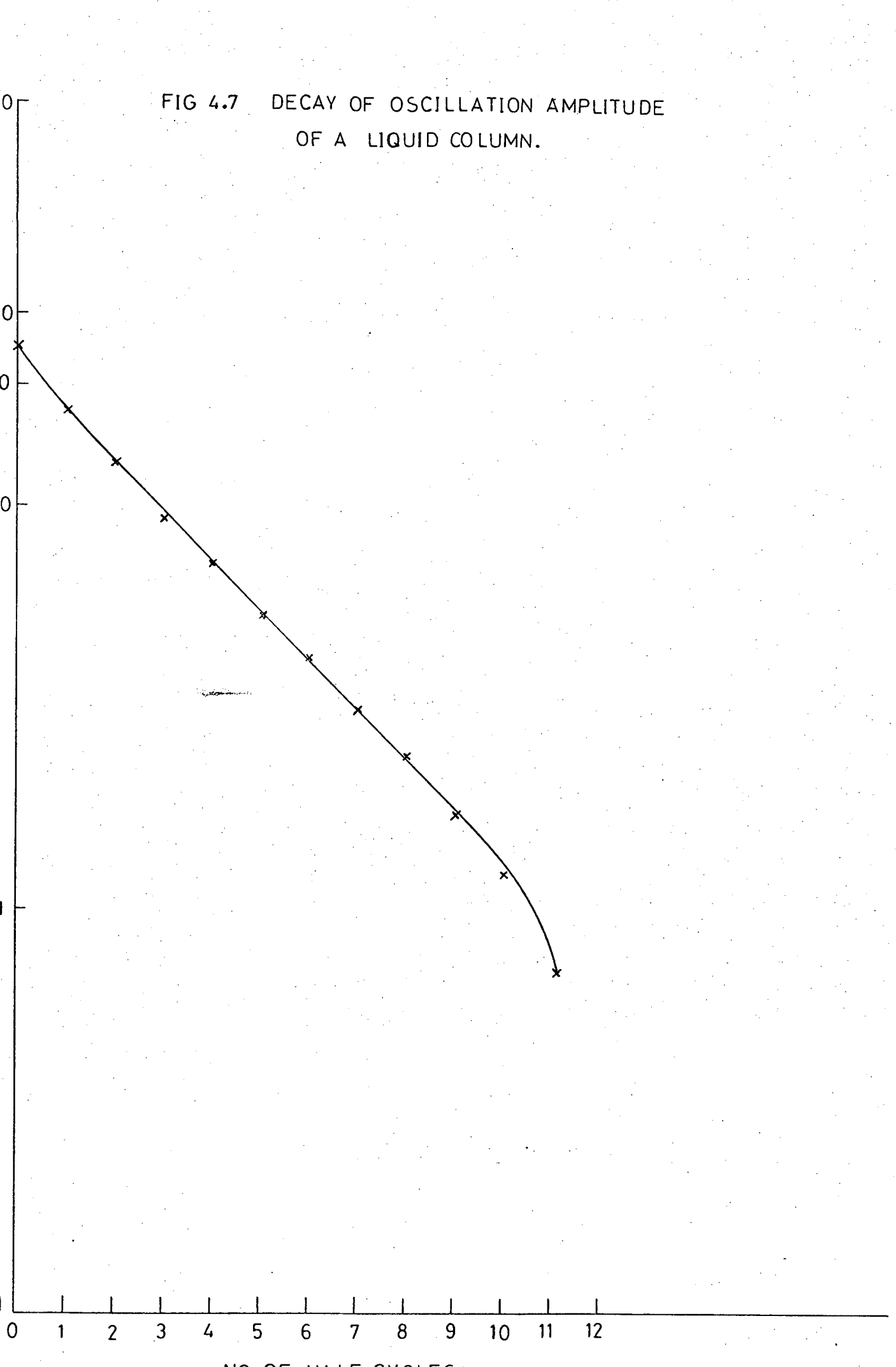
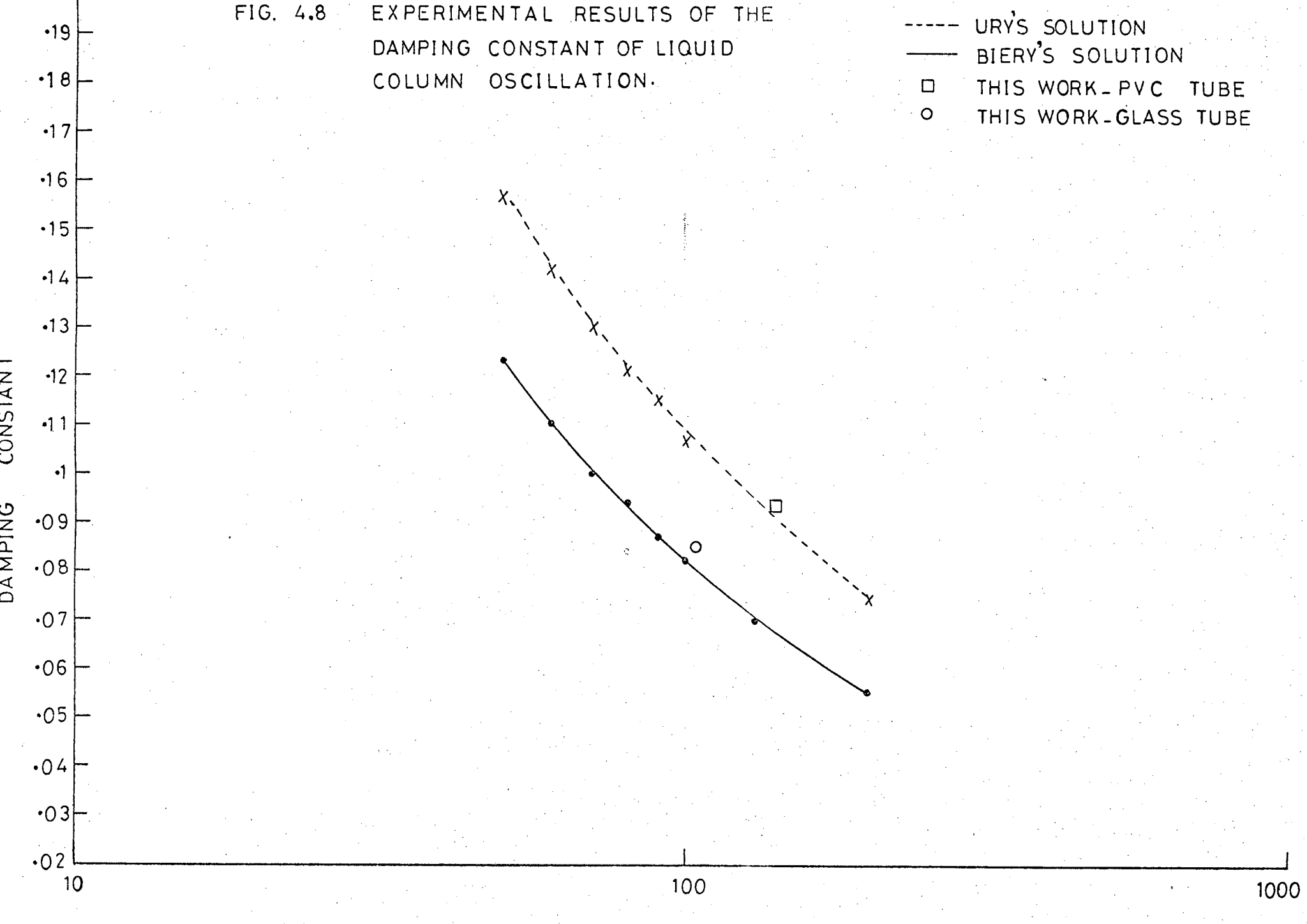


FIG. 4.8 EXPERIMENTAL RESULTS OF THE DAMPING CONSTANT OF LIQUID COLUMN OSCILLATION.



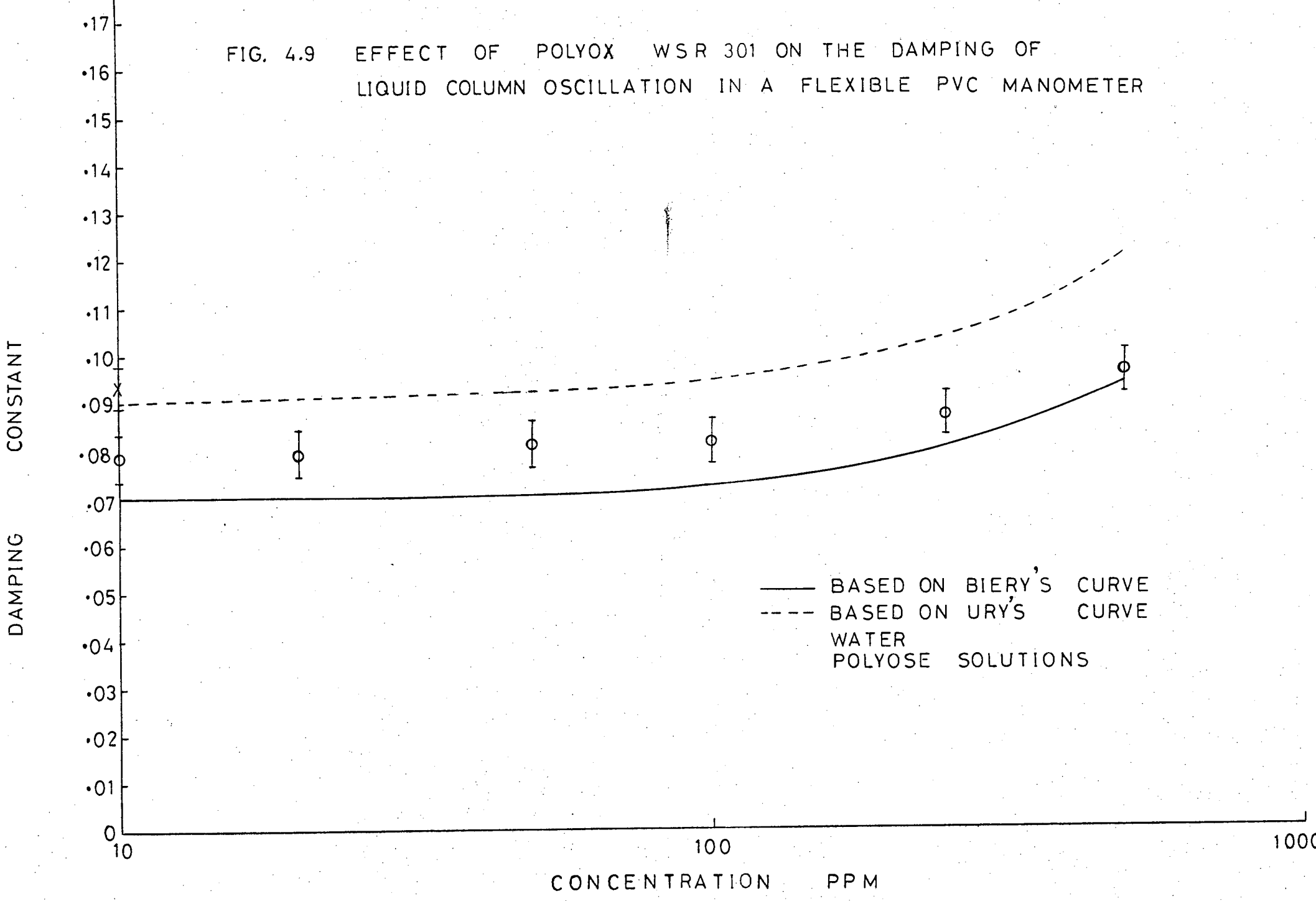


FIG. 4.9 EFFECT OF POLYOX WSR 301 ON THE DAMPING OF LIQUID COLUMN OSCILLATION IN A FLEXIBLE PVC MANOMETER
DAMPING CONSTANT
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·16
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0
10
100
1000
CONCENTRATION PPM
BASED ON BIERY'S CURVE
BASED ON URY'S CURVE
WATER
POLYOSE SOLUTIONS

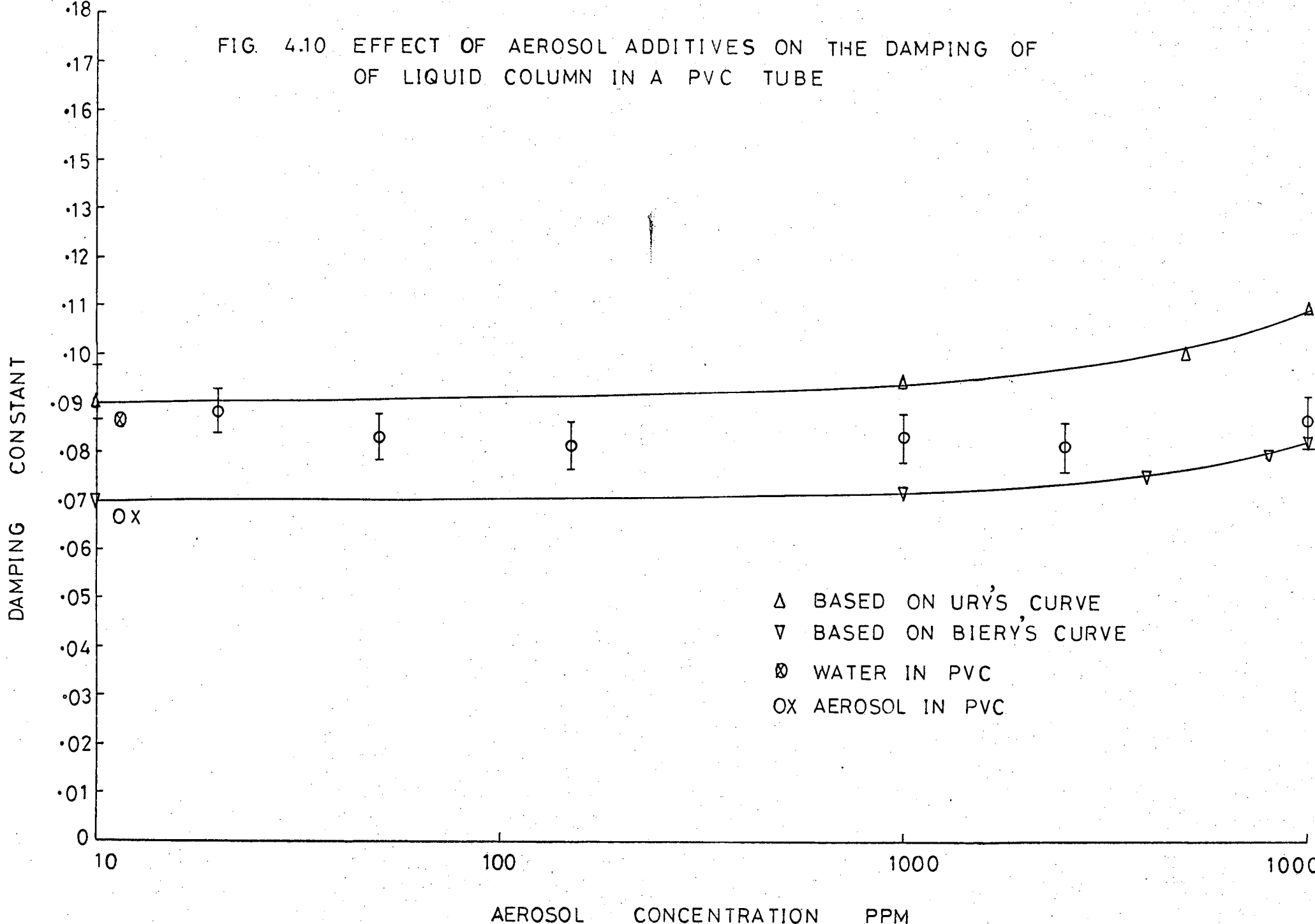
FIG. 4.10 EFFECT OF AEROSOL ADDITIVES ON THE DAMPING OF OF LIQUID COLUMN IN A PVC TUBE
DAMPING CONSTANT
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·12
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·08
·07
·06
·05
·04
·03
·02
·01
0
OX
10
100
1000
1000
AEROSOL CONCENTRATION PPM
Δ BASED ON URY'S CURVE
∇ BASED ON BIERY'S CURVE
⊗ WATER IN PVC
OX AEROSOL IN PVC

FIG 4.11 VISCOSITY OF AEROSOL SOLUTIONS

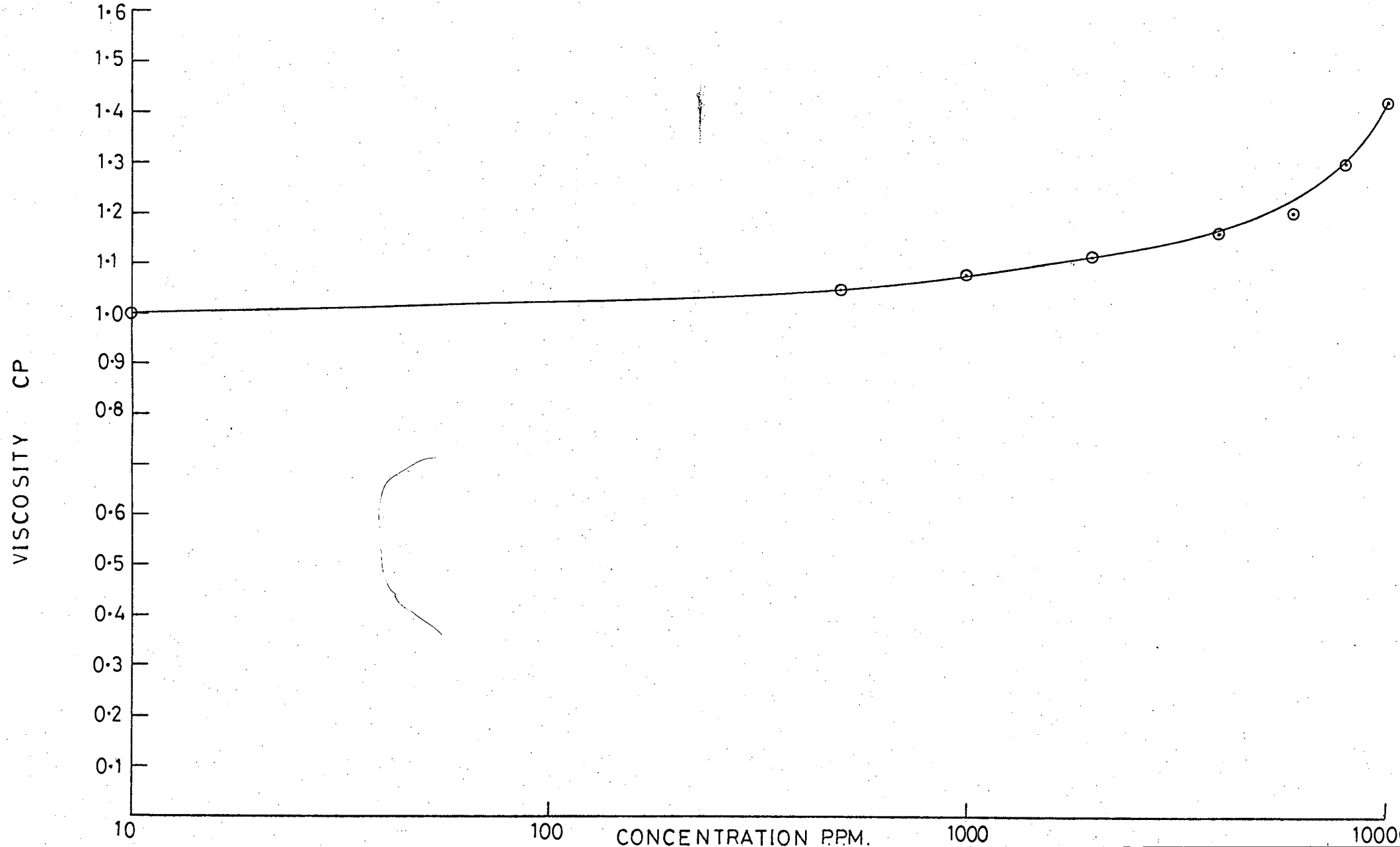


FIG. 4.12 EFFECT OF POLYOXE WSR 301 ADDITIVES ON THE DAMPING OF LIQUID COLUMN OSCILLATION IN A COATED FLEXIBLE P.V.C MANOMETER

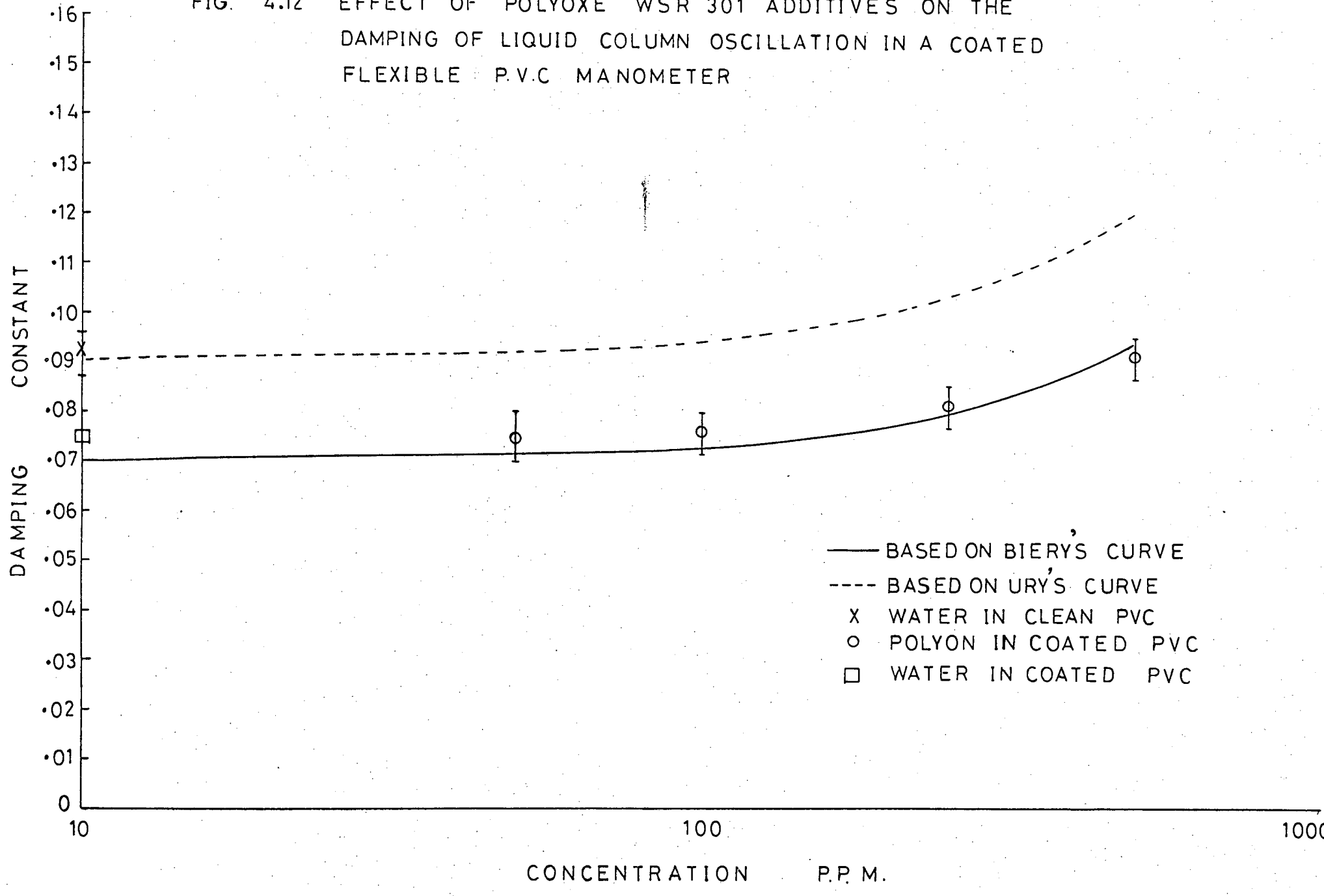


FIG. 4.13 EFFECT OF AEROSOL ON THE DAMPING OF LIQUID COLUMN IN A GLASS MANOMETER

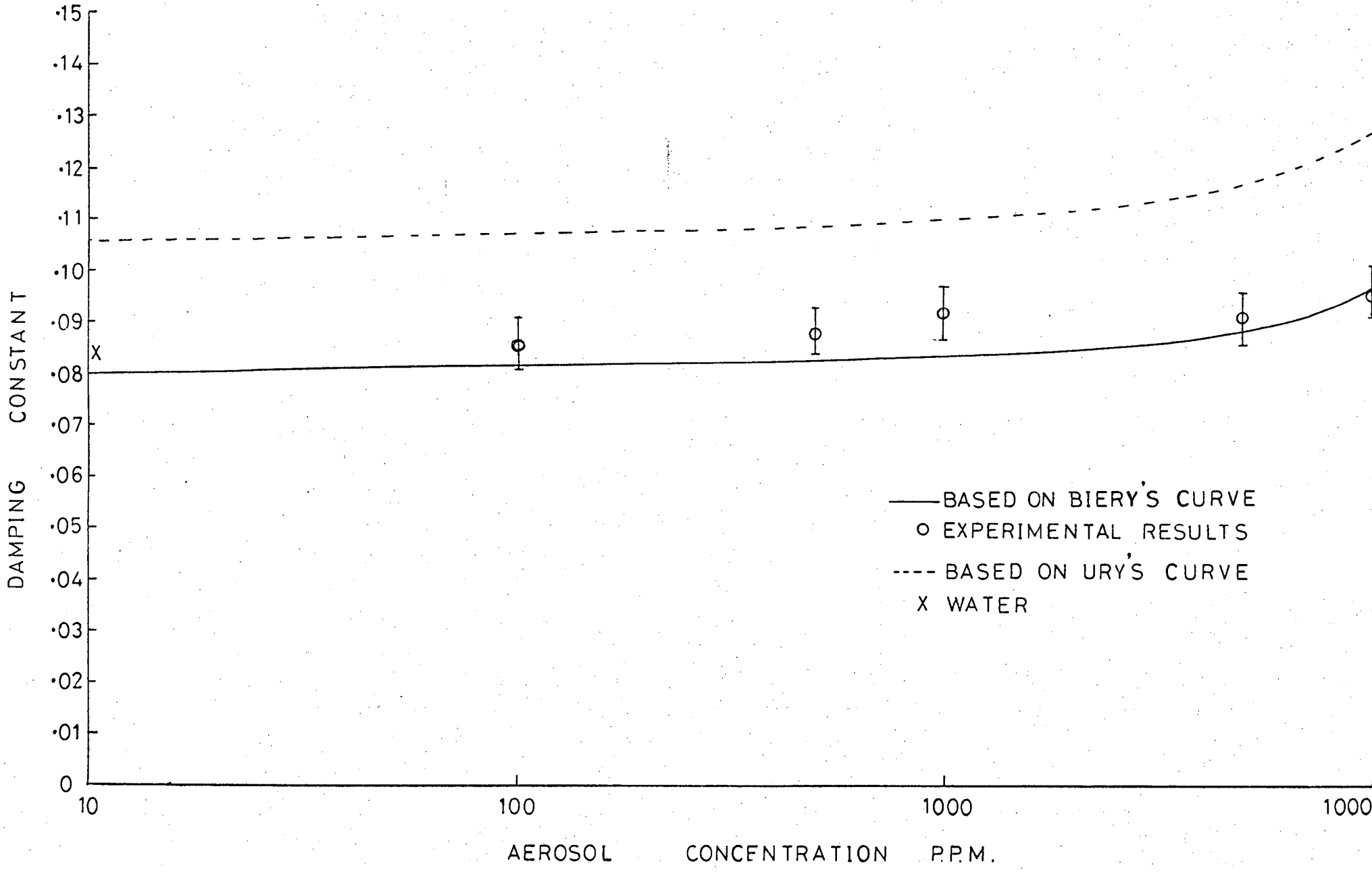


FIG. 4.14 EFFECT OF POLYOX WSR-301 ADDITIVE ON THE DAMPING OF LIQUID COLUMN OSCILLATIONS IN A GLASS MANOMETER.

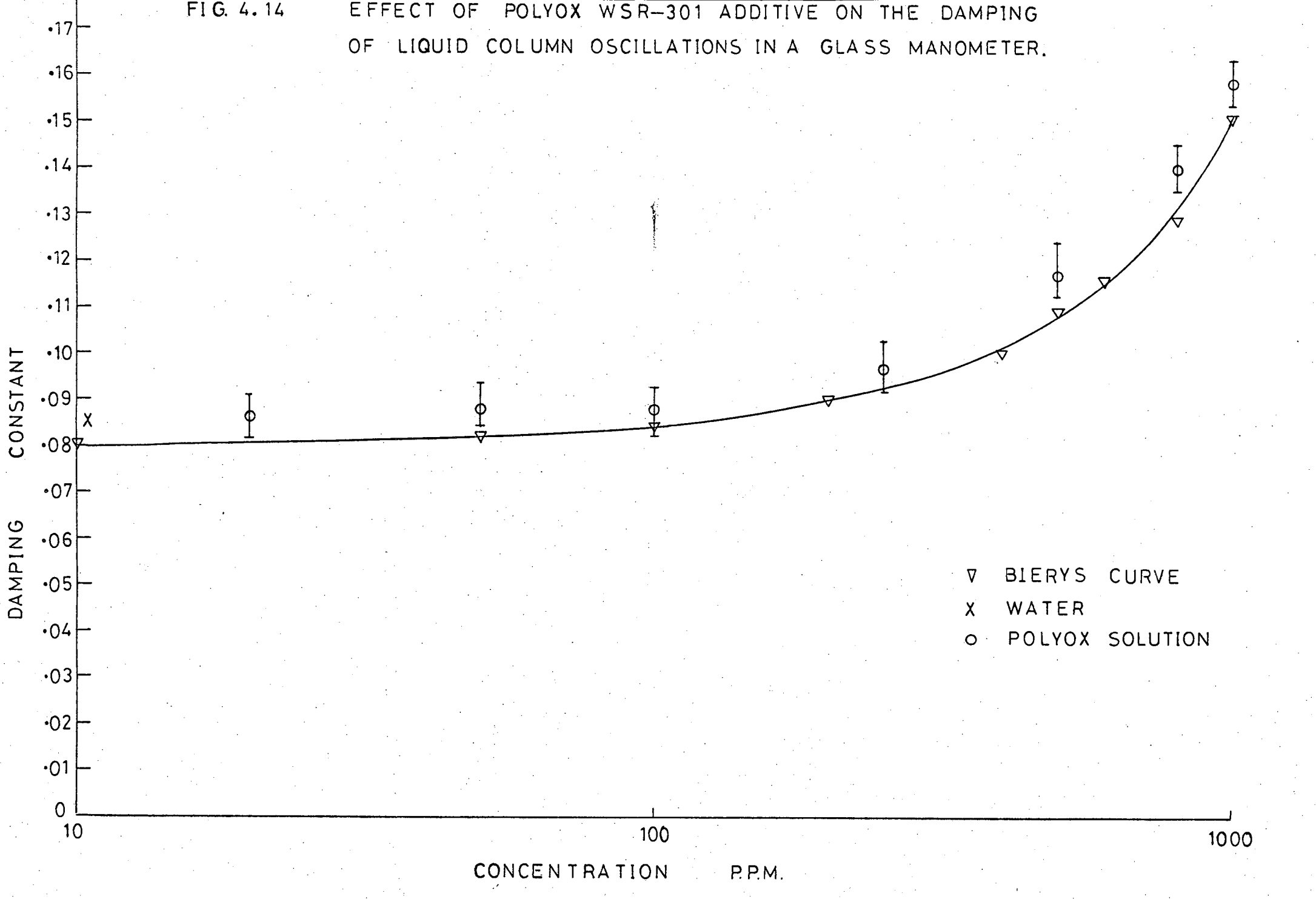
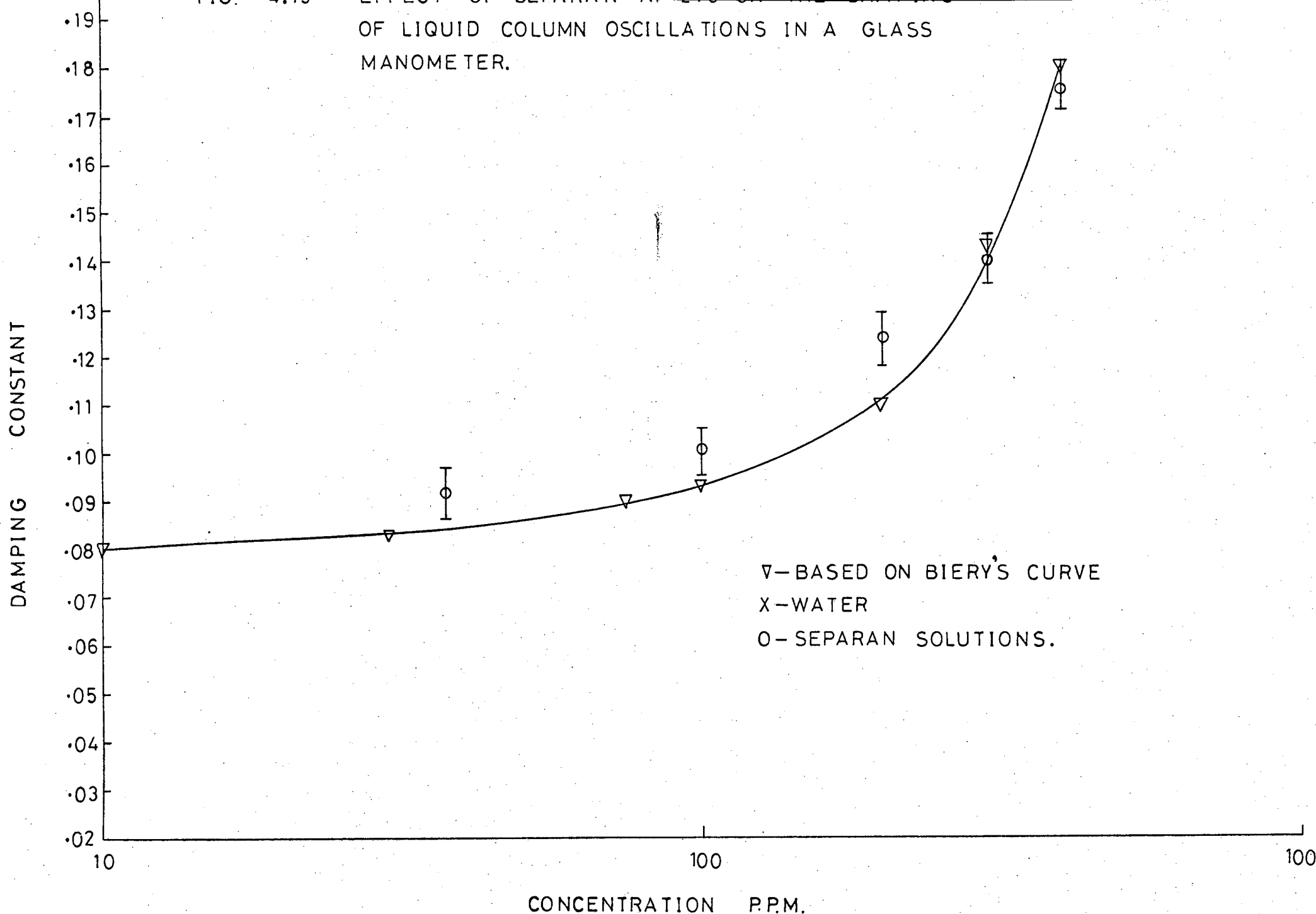


FIG. 4.13 EFFECT OF SEPARAN ON DAMPING OF LIQUID COLUMN OSCILLATIONS IN A GLASS MANOMETER.



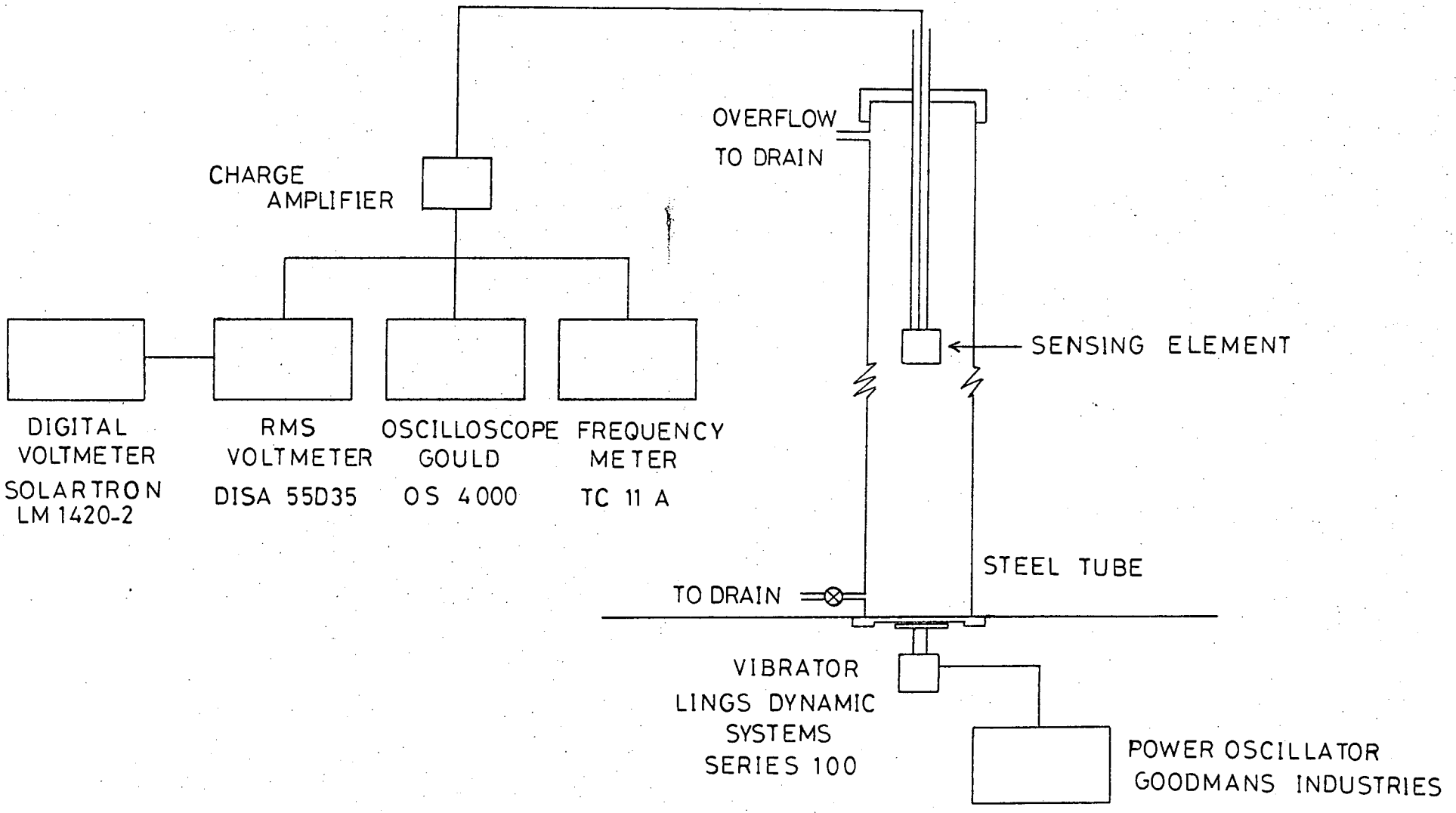


FIG. 5.1 SOUND VELOCITY MEASUREMENTS EXPERIMENTAL SET-UP.

FIG 5.2 MEASUREMENT OF SOUND VELOCITY
BY STANDING WAVE METHOD. FREQUENCY 1450 HZ.

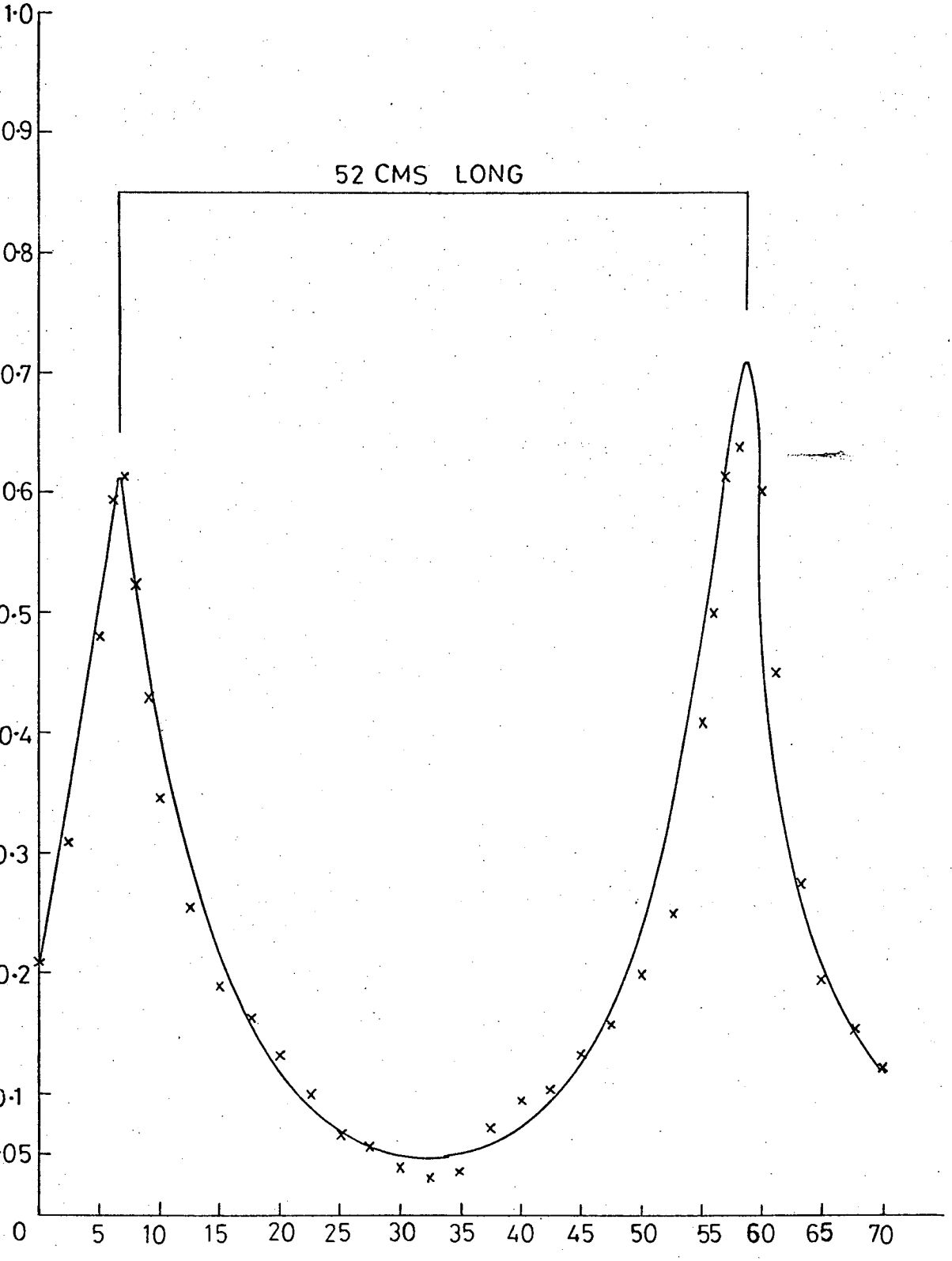
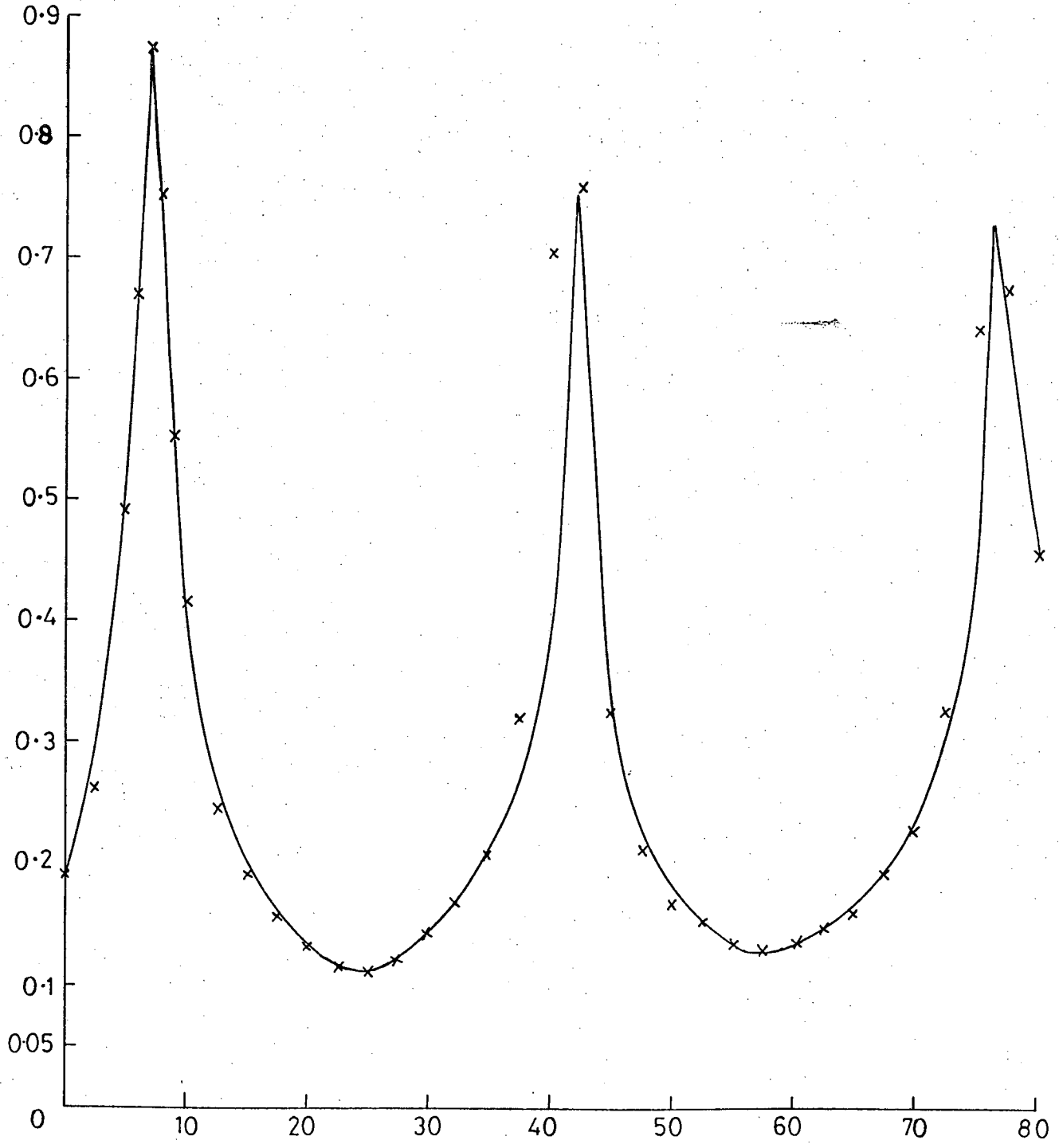


FIG. 5.3 MEASUREMENT OF SOUND VELOCITY BY STANDING WAVE METHOD. FREQUENCY 2100 HZ.

WATER.



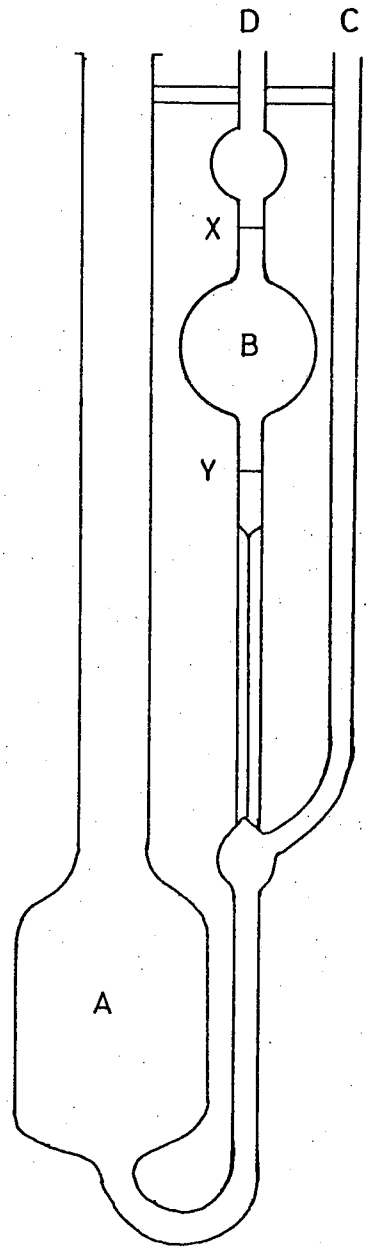
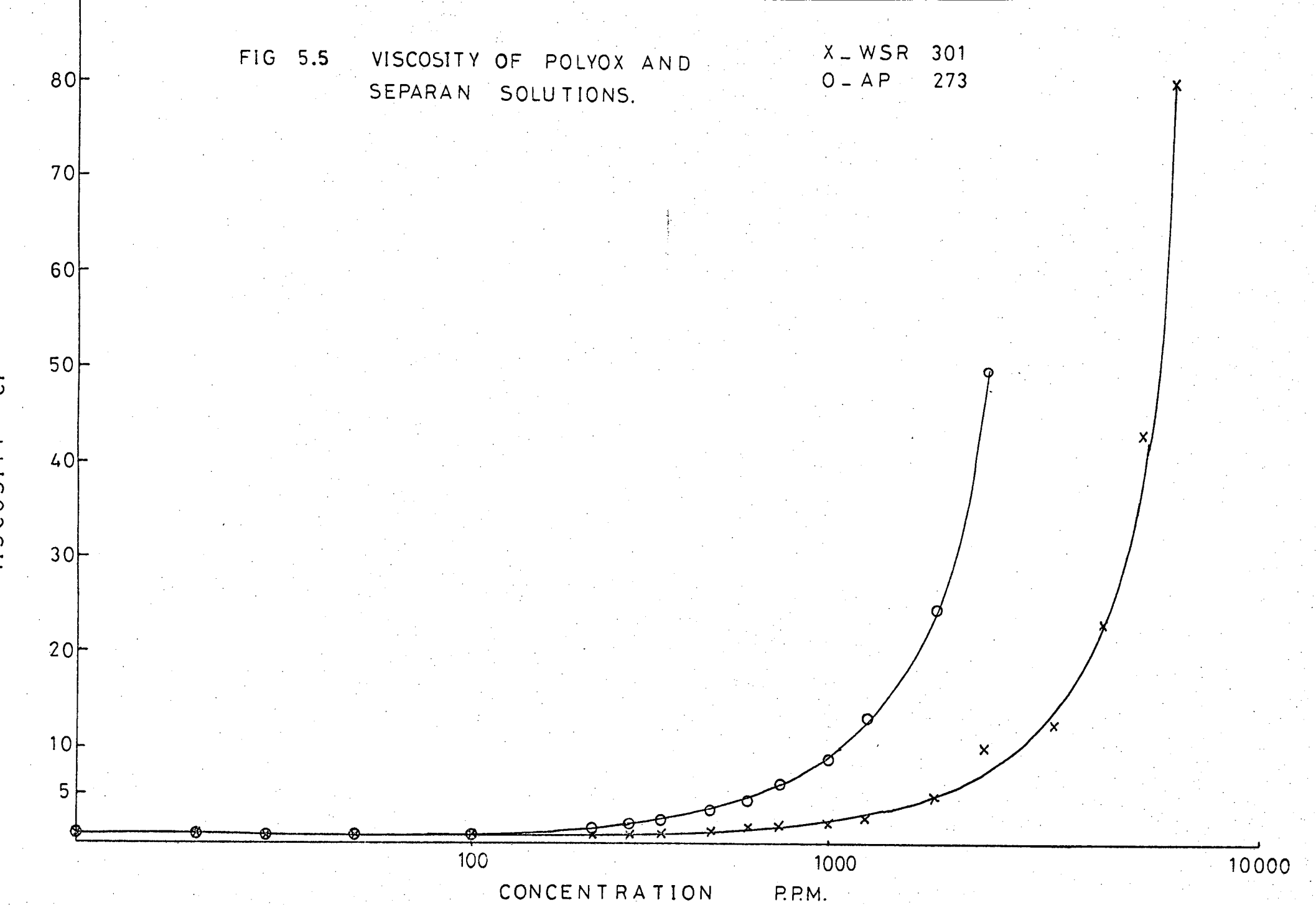


FIG. 5.4 UBBELOHDE VISCOMETER

FIG 5.5 VISCOSITY OF POLYOX AND SEPARAN SOLUTIONS.

X - WSR 301
O - AP 273



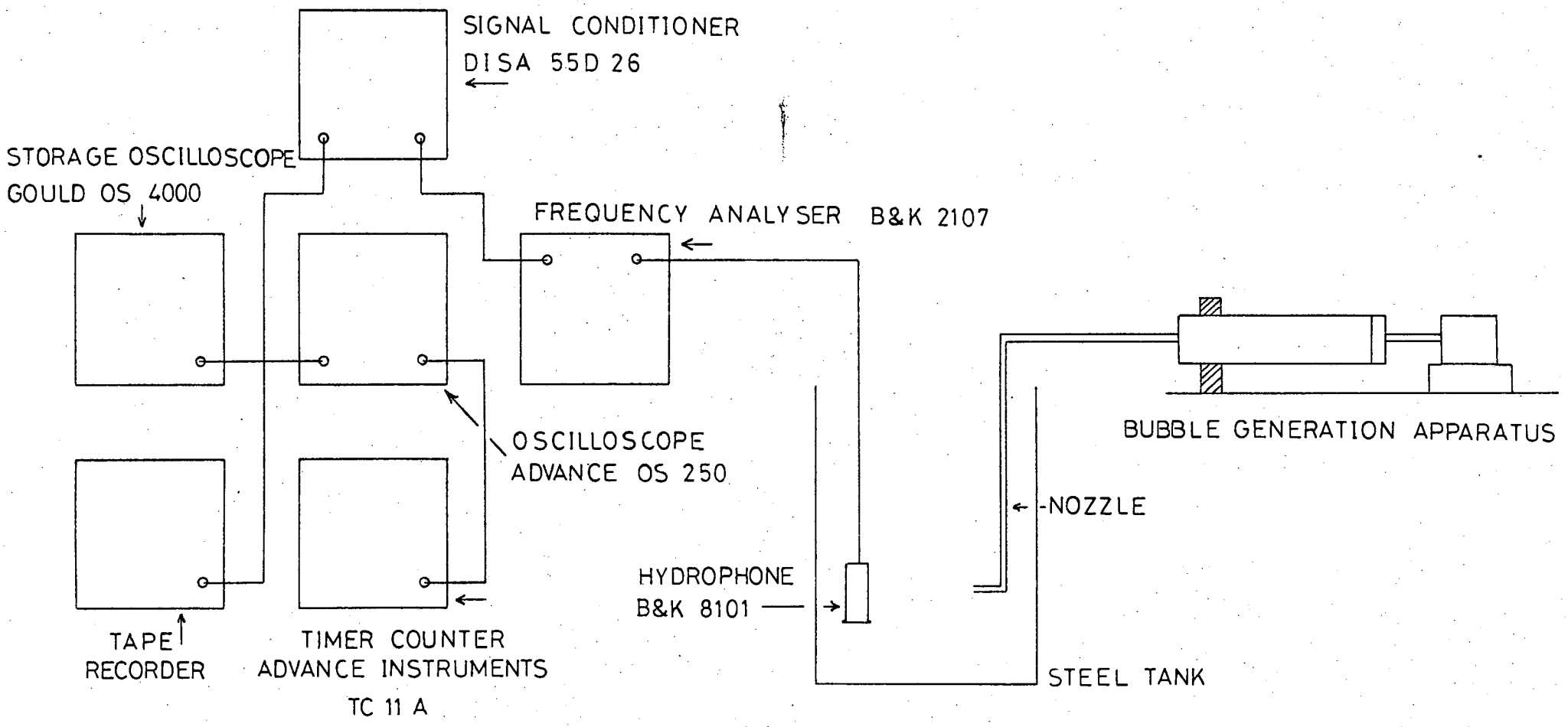


FIG. 6.1 BUBBLE GENERATION EXPERIMENTAL SET-UP.

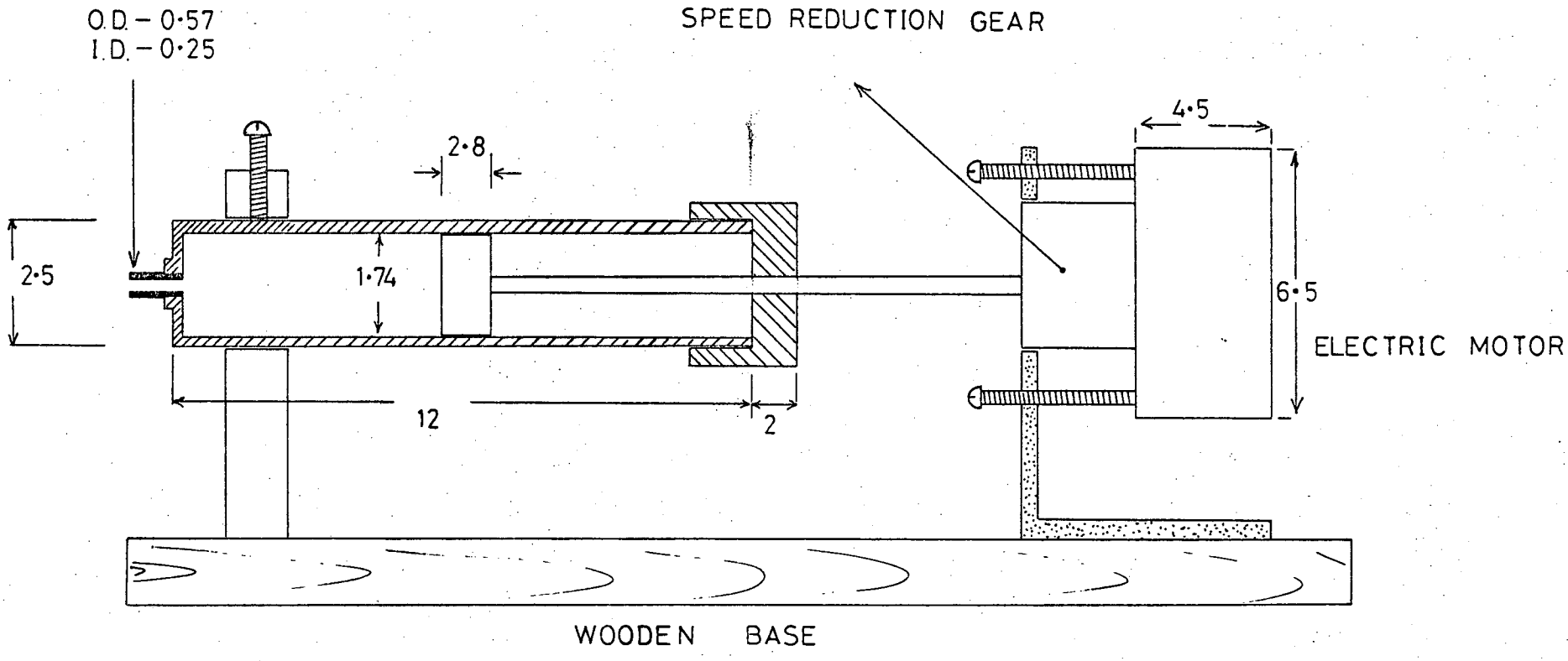
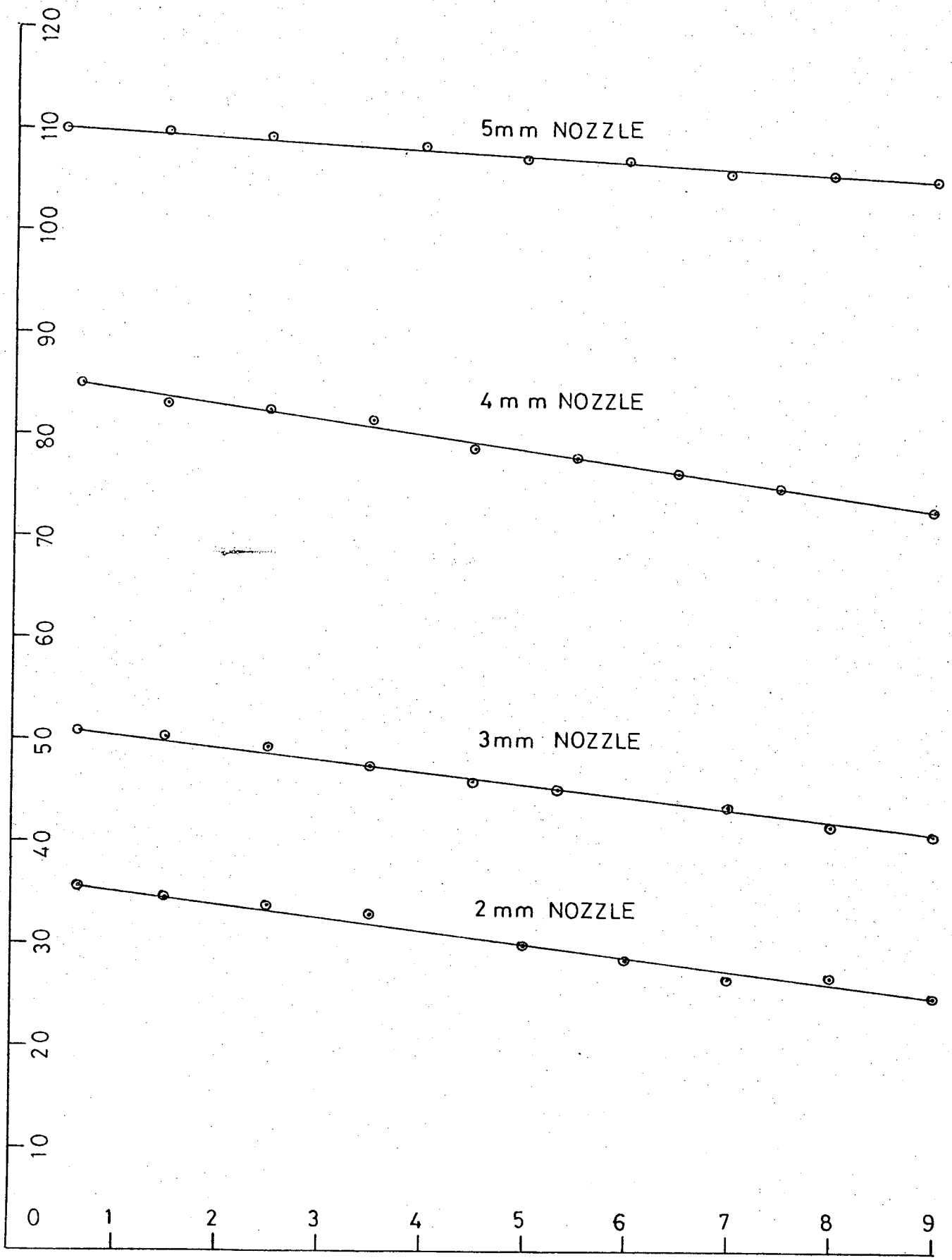


FIG 6.2 BUBBLE GENERATOR APPARATUS. SECTIONAL VIEW .

FIG. 6.3 RATE OF CHANGE OF AVERAGE BUBBLE VOLUME



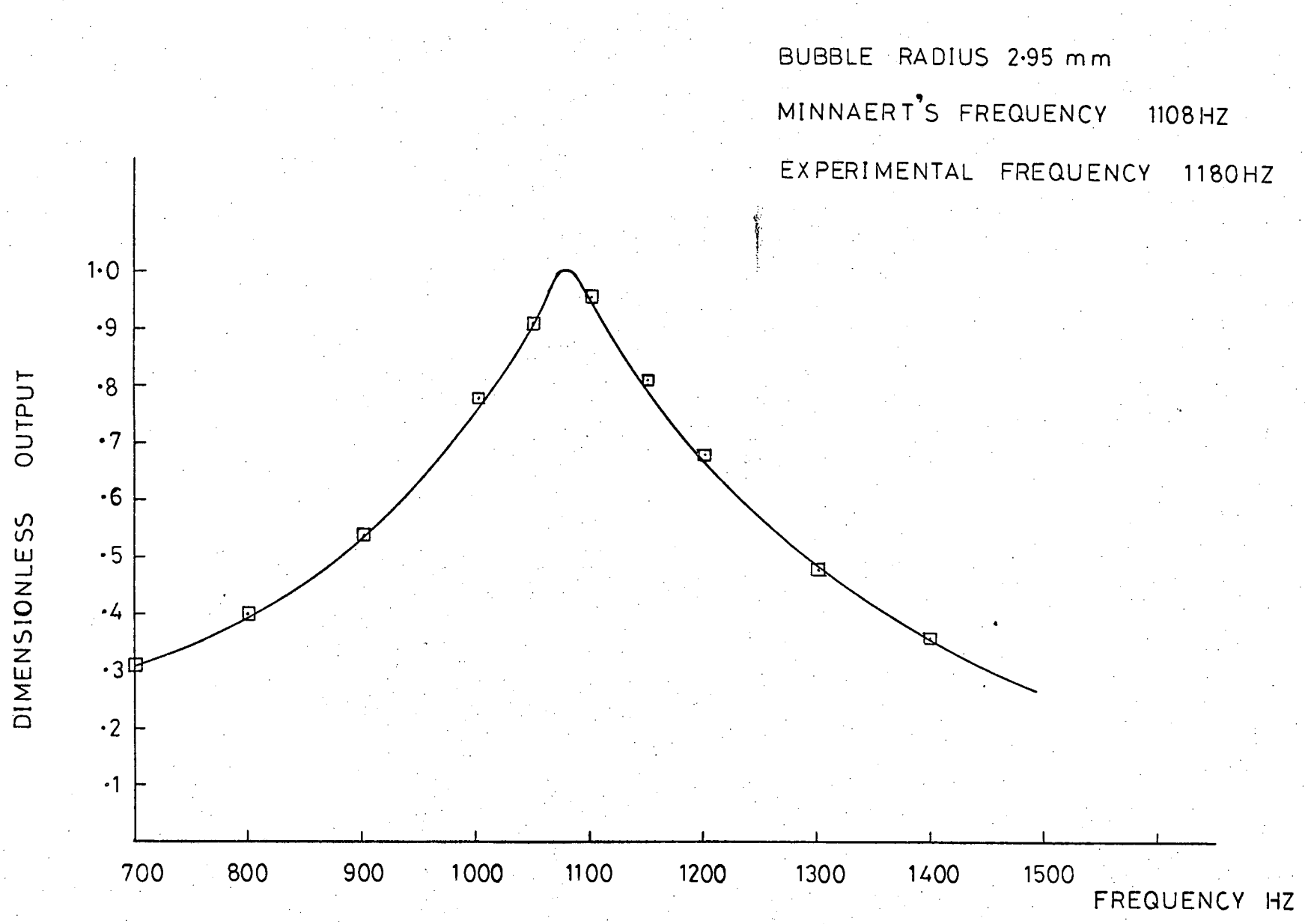


FIG. 6.4 SPECTRUM ANALYSIS OF BUBBLE SIGNAL

FIG. 8.1 EFFECT OF PROTRUDING RATIO OF THE NOZZLE ON THE DAMPING OF BUBBLE PULSATION.

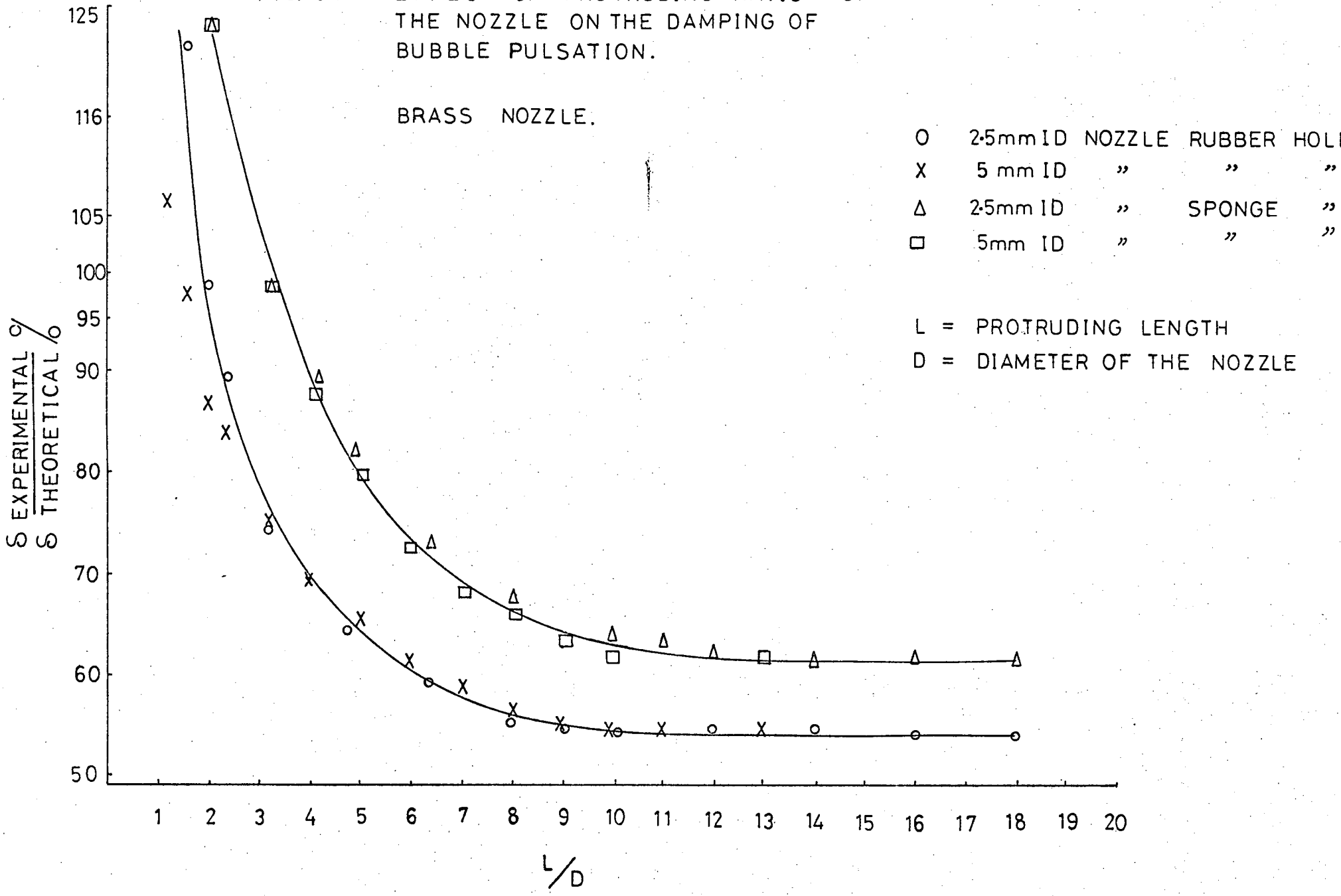
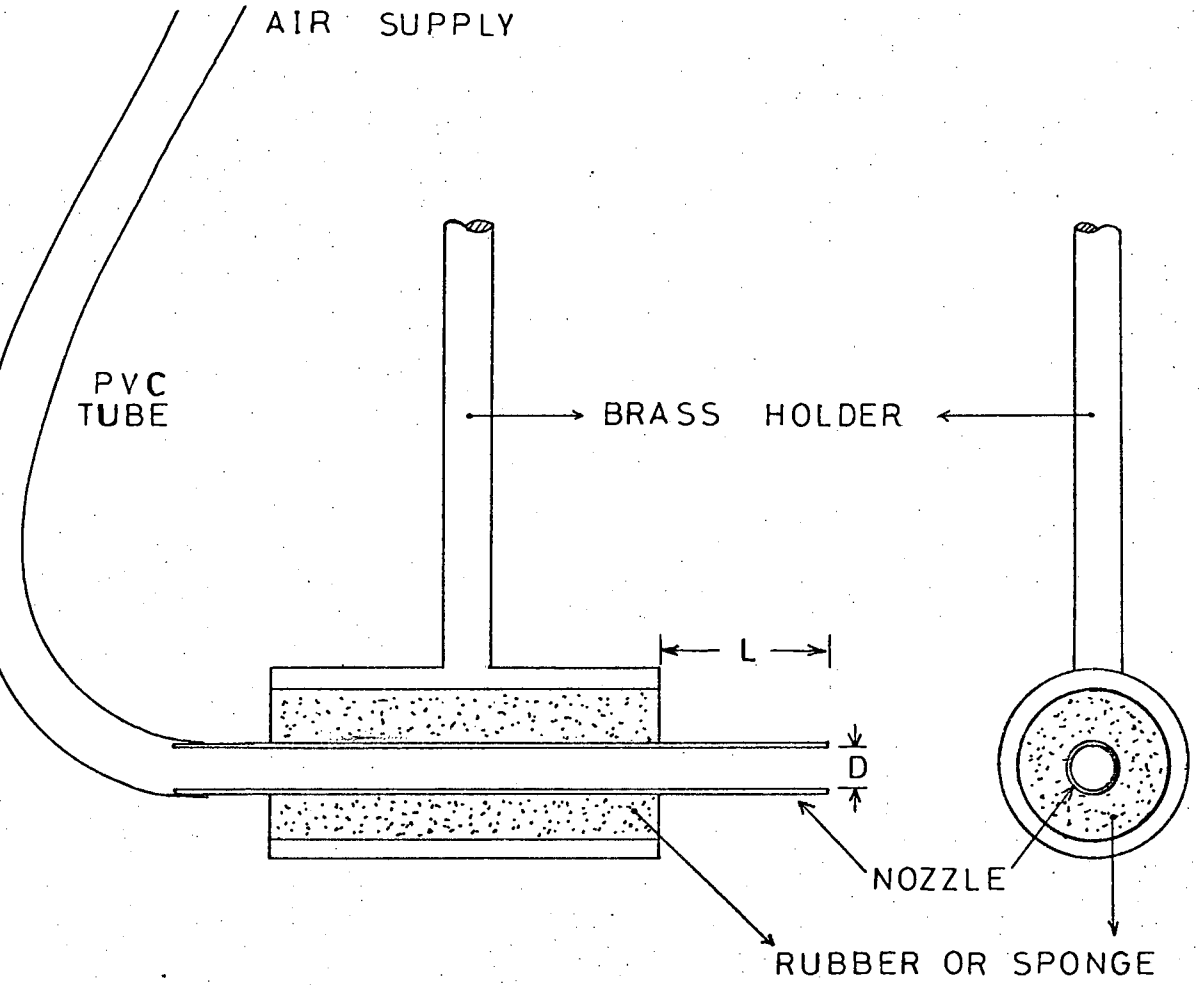


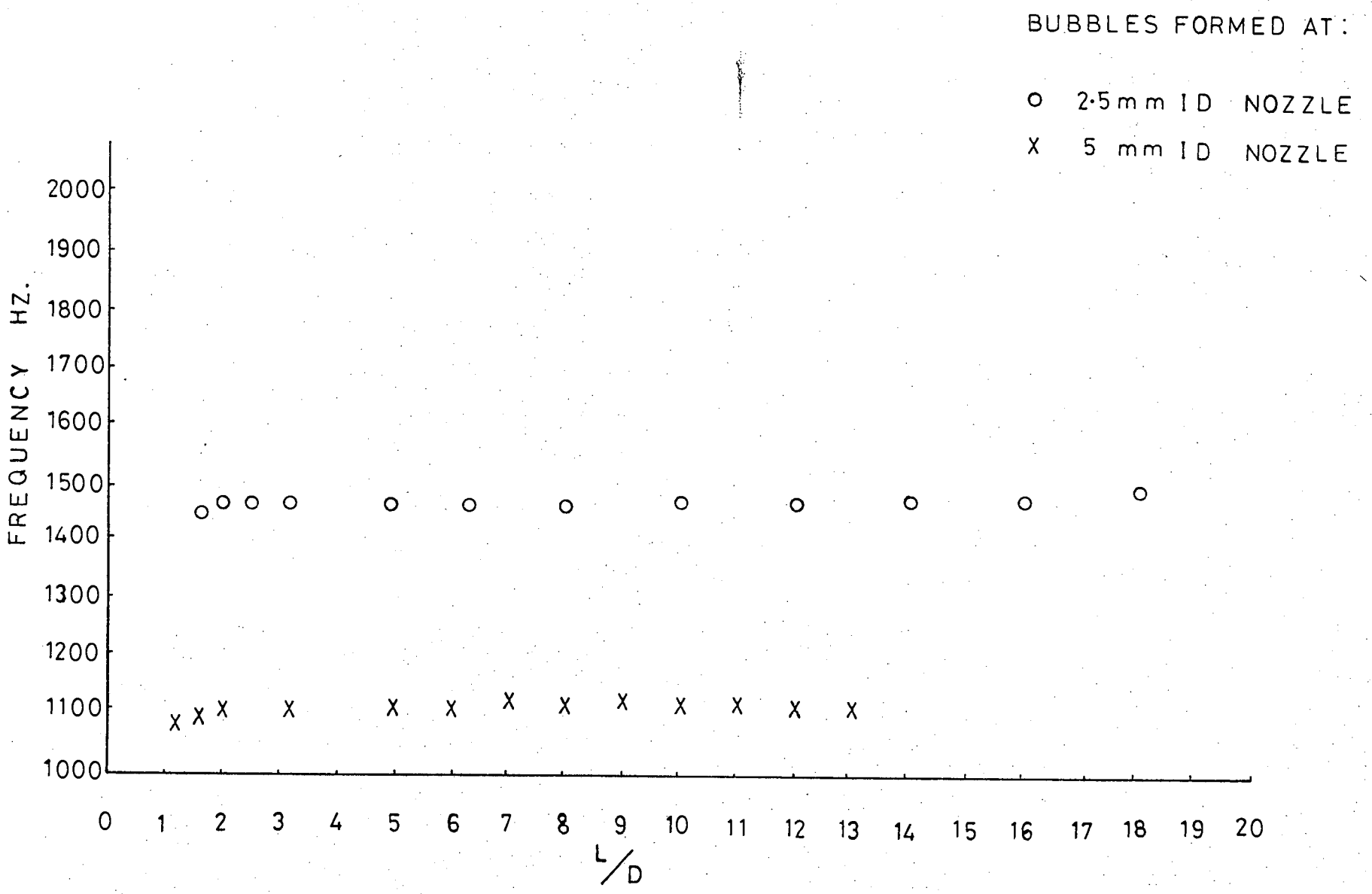
FIG. 8.1a A SCHEMATIC DIAGRAM OF THE NOZZLE AND HOLDER



PROTRUDING RATIO L/D

D = INTERNAL DIAMETER.

FIG. 8.2 EFFECT OF THE PROTRUDING RATIO ON BUBBLE FREQUENCY



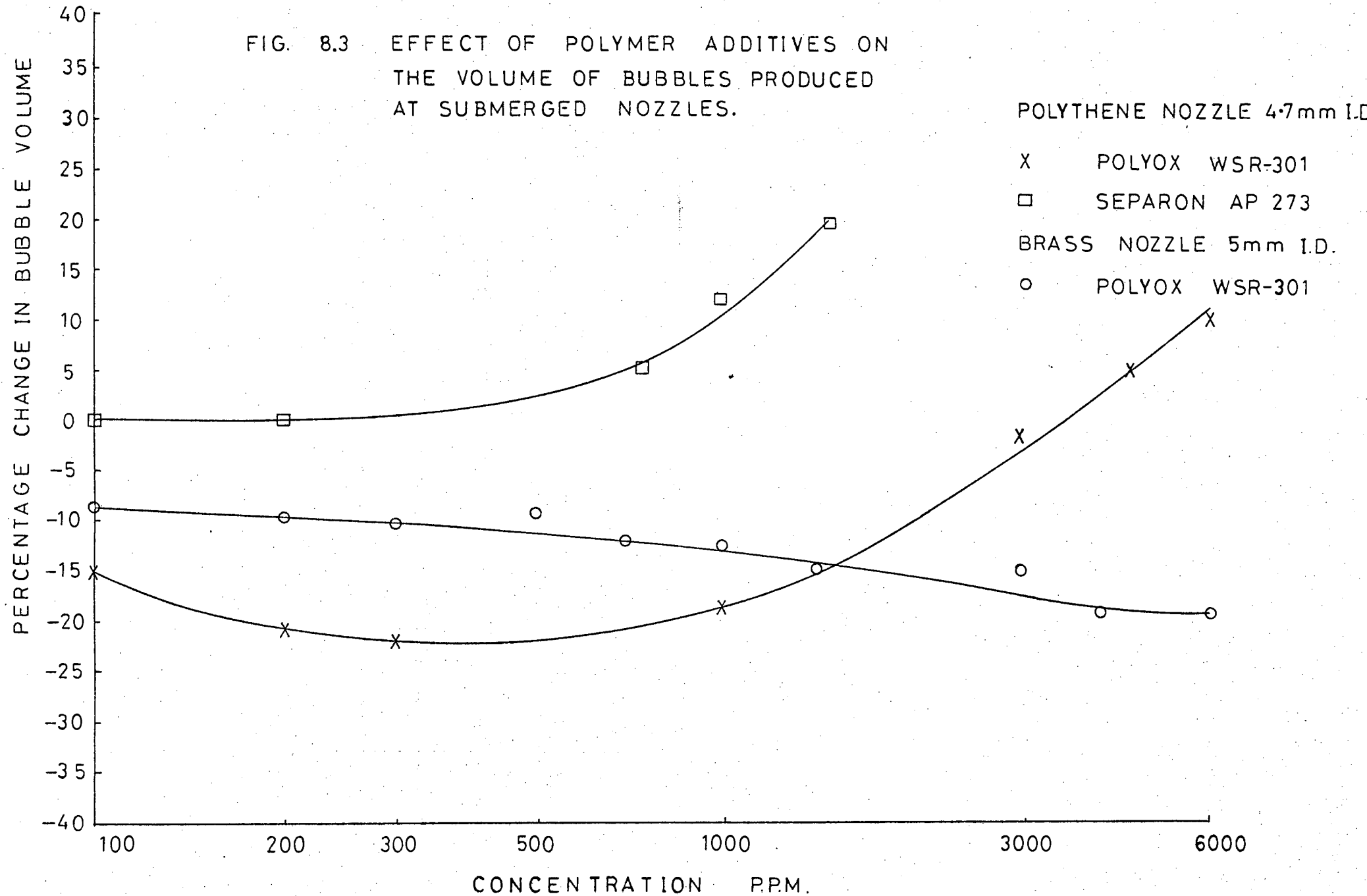


FIG. 8.3 EFFECT OF POLYMER ADDITIVES ON THE VOLUME OF BUBBLES PRODUCED AT SUBMERGED NOZZLES.
POLYTHENE NOZZLE 4·7mm I.D
X POLYOX WSR-301
□ SEPARON AP 273
BRASS NOZZLE 5mm I.D.
○ POLYOX WSR-301
PERCENTAGE CHANGE IN BUBBLE VOLUME
40
35
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
-40
100
200
300
500
1000
3000
6000
CONCENTRATION P.P.M.

FIG. 8.4 EFFECT OF POLYMER ADDITIVES ON BUBBLE PULSATING FREQUENCY

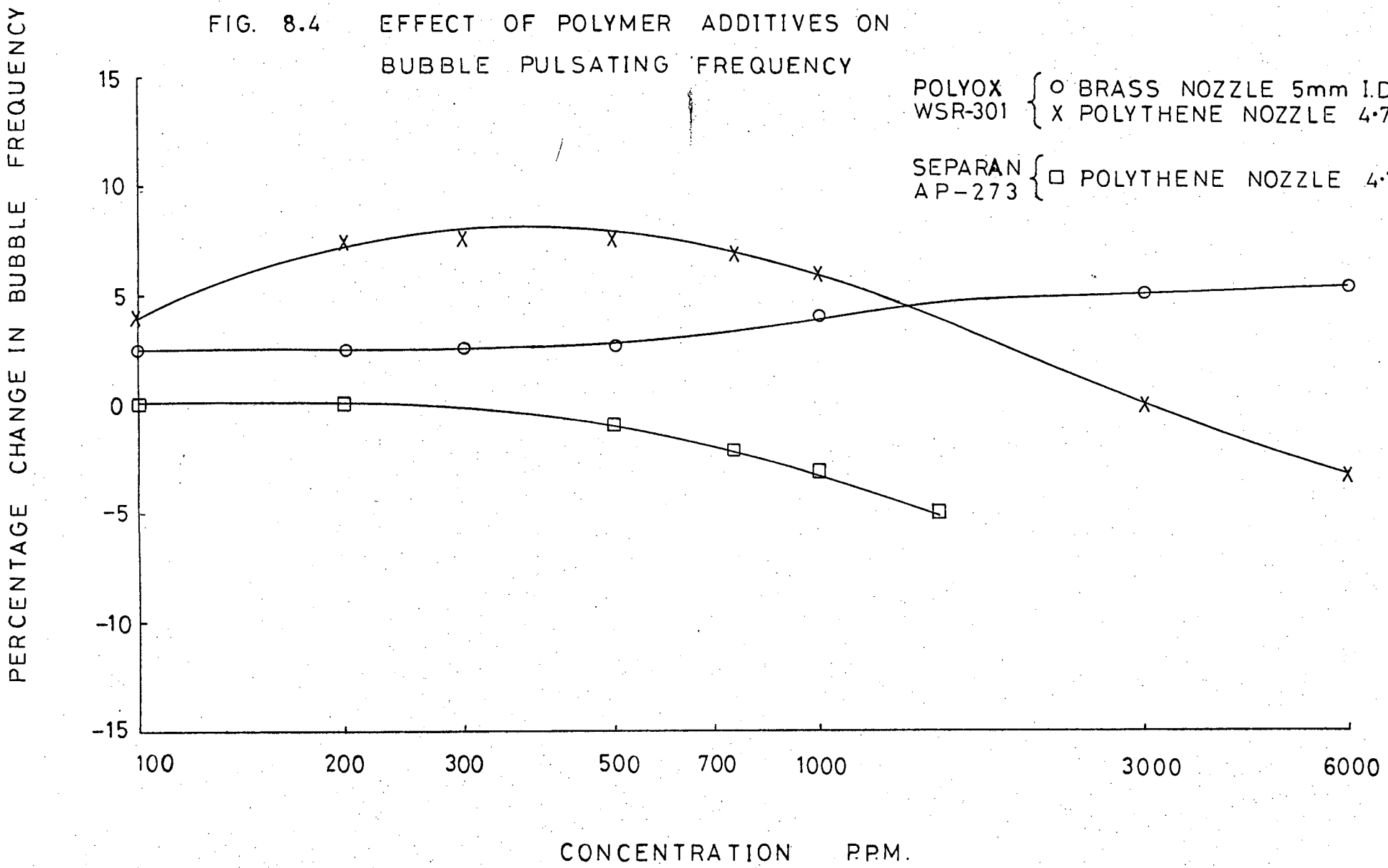


FIG. 8.5 EFFECT OF LIQUID RELAXATION TIME ON THE FREQUENCY OF BUBBLE PULSATION.

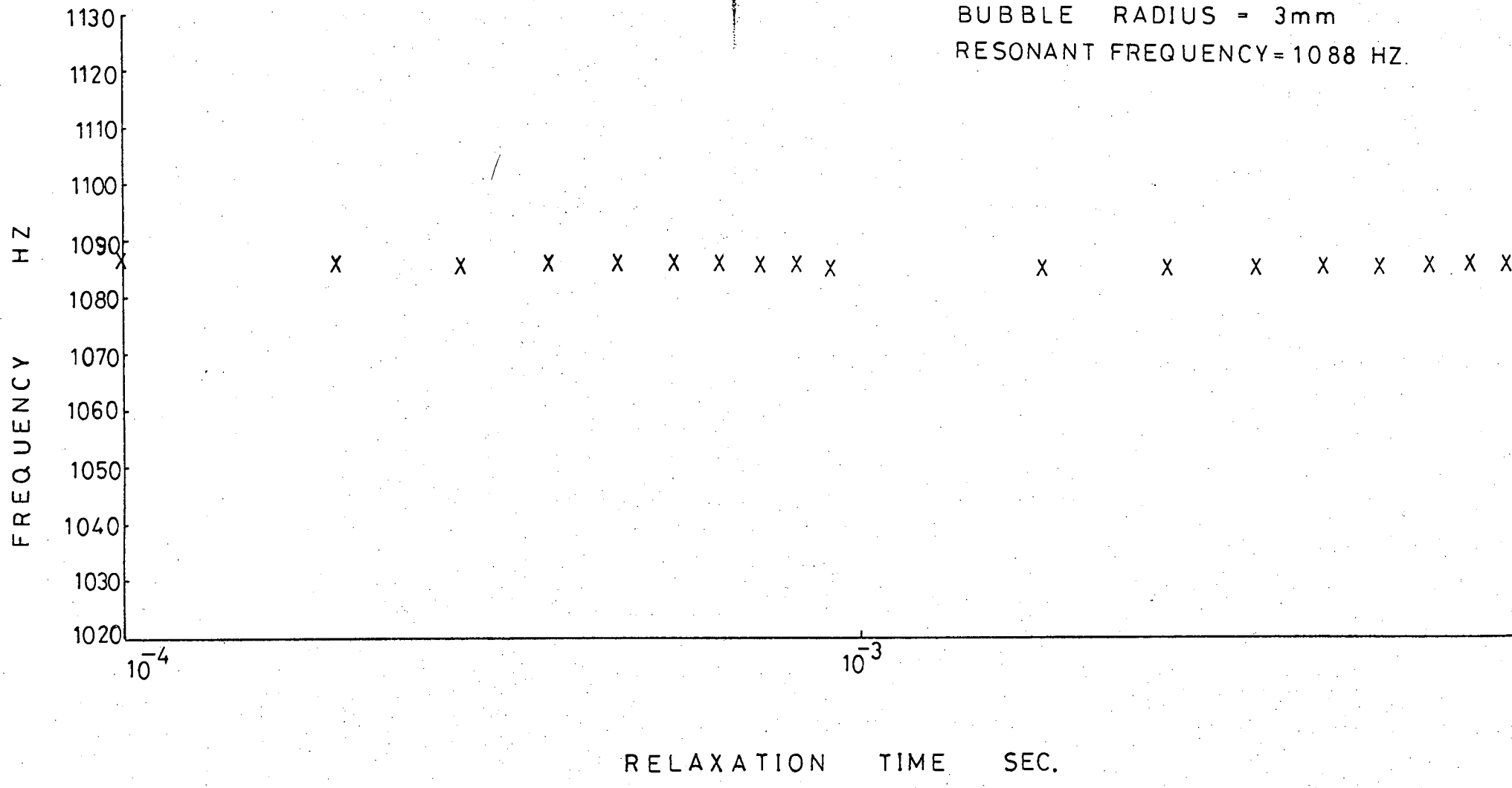


FIG. 8.6 NOISE INTENSITY Vs. CONCENTRATION

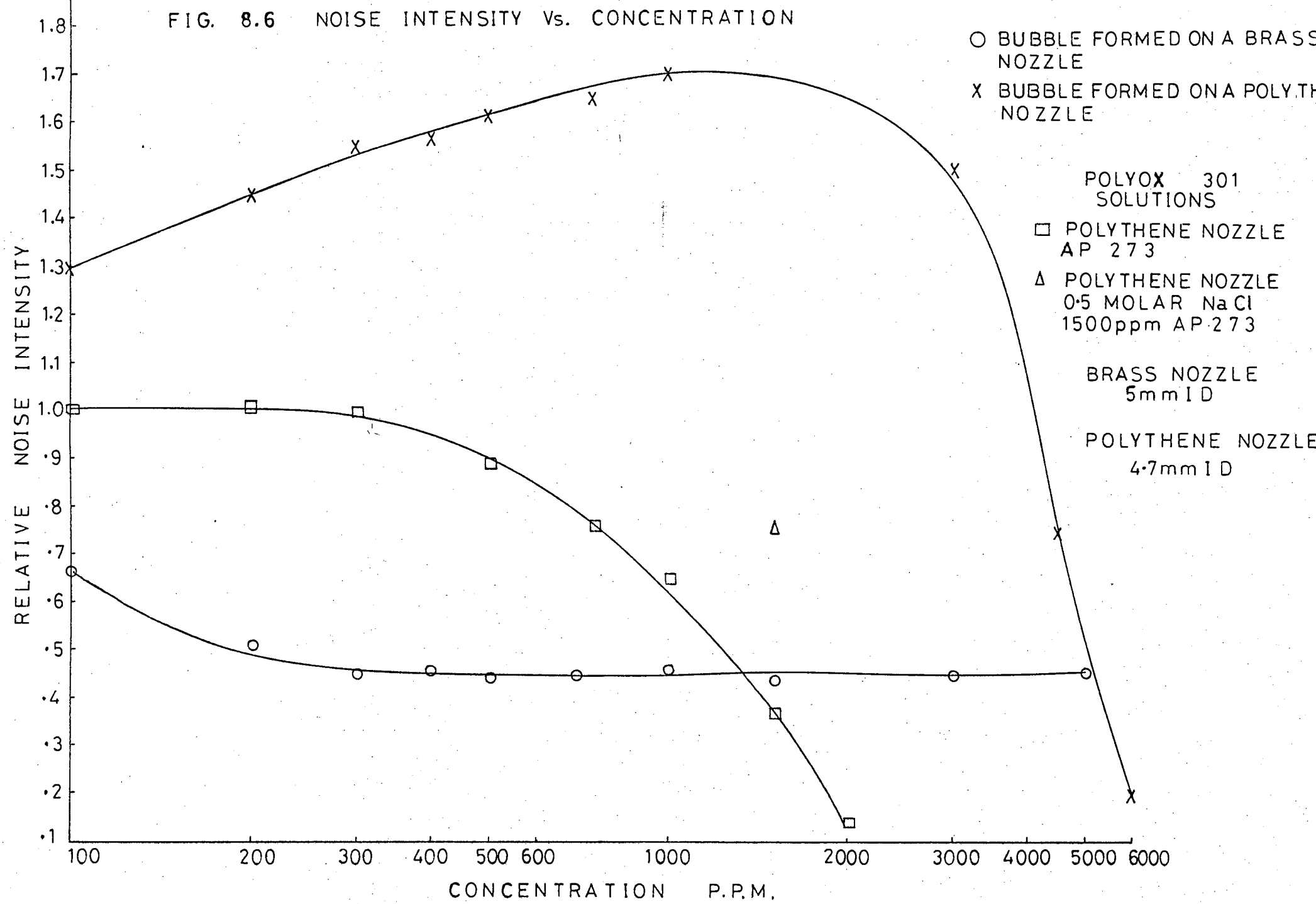
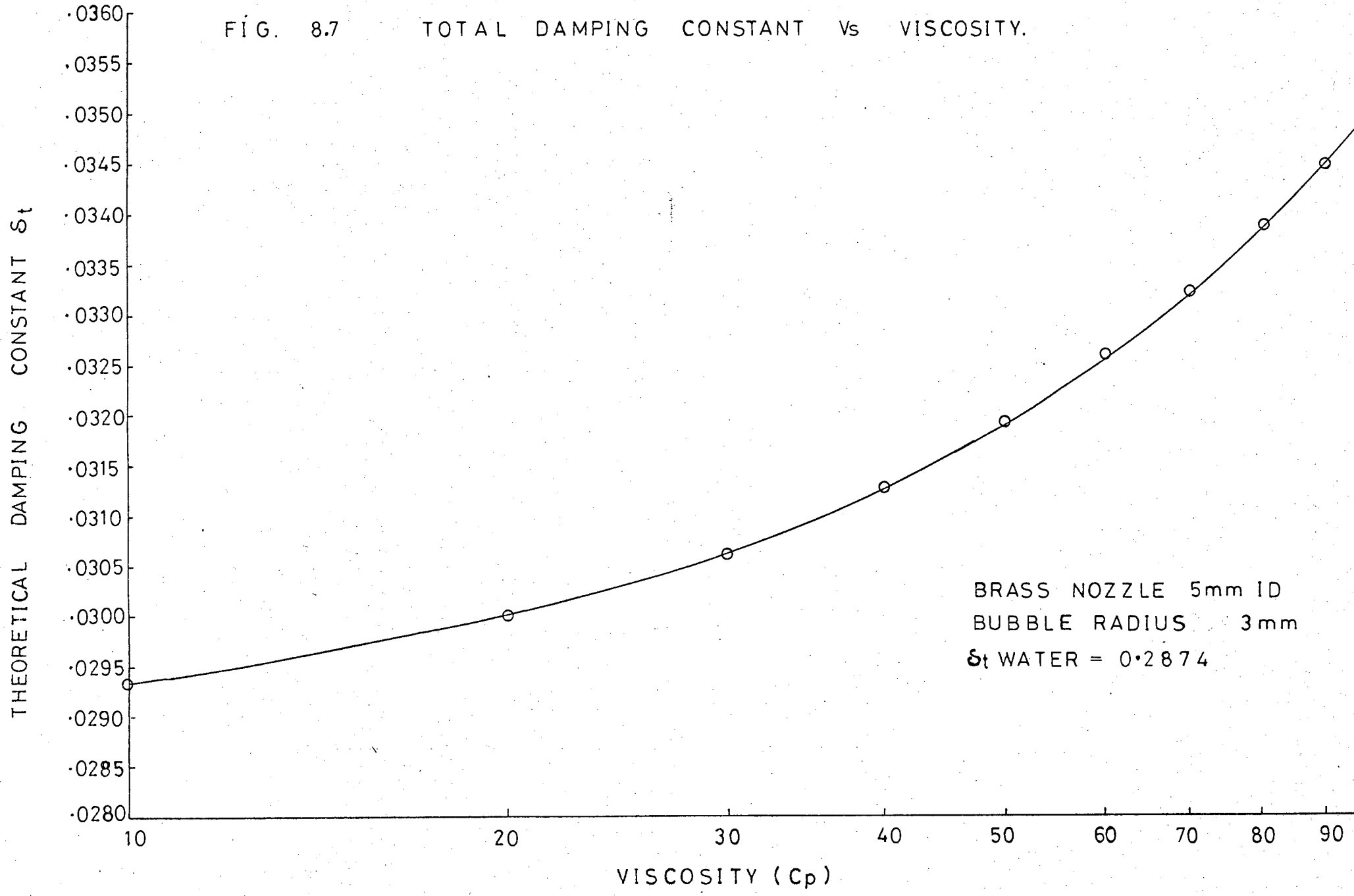


FIG. 8.7 TOTAL DAMPING CONSTANT Vs VISCOSITY.



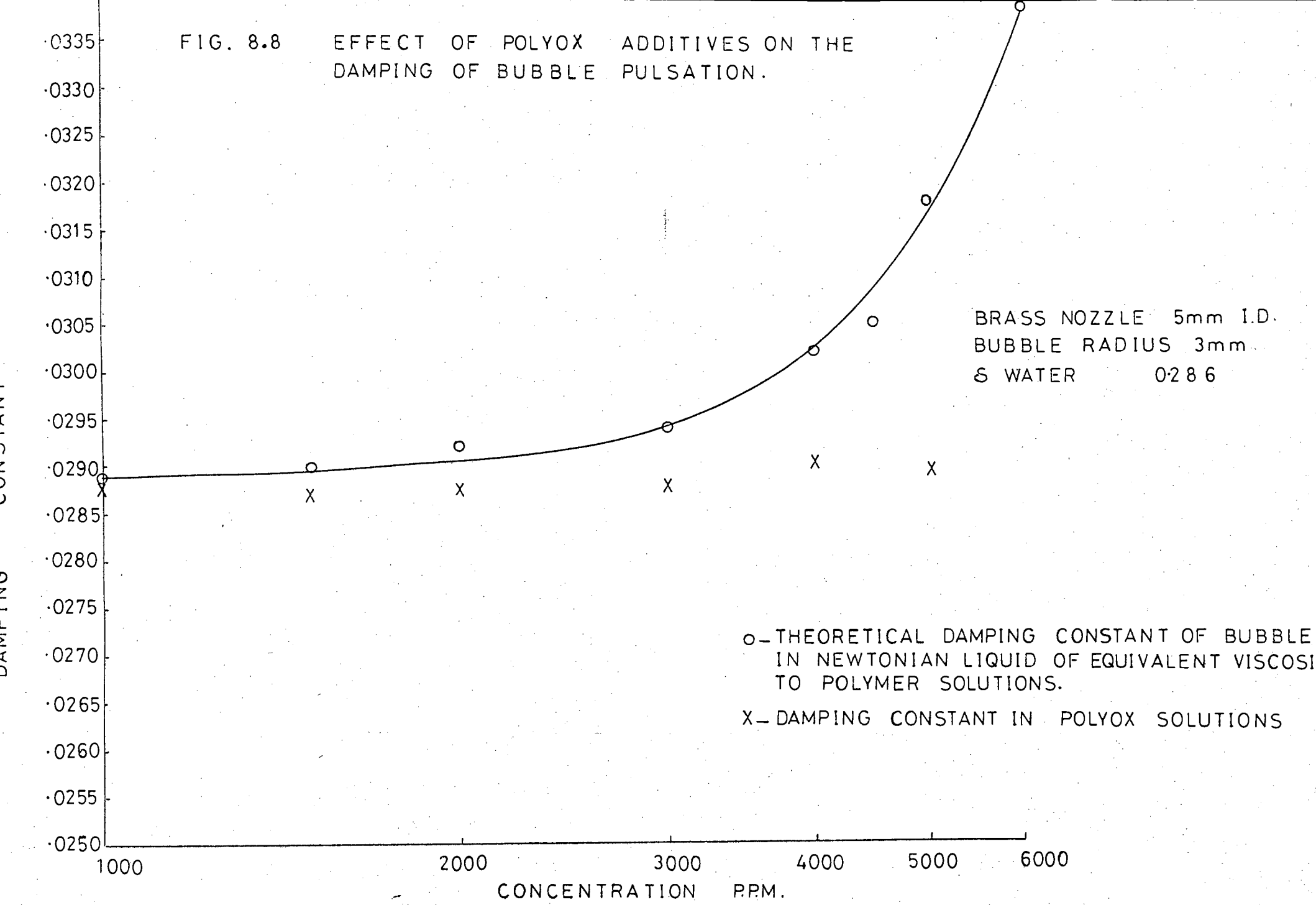
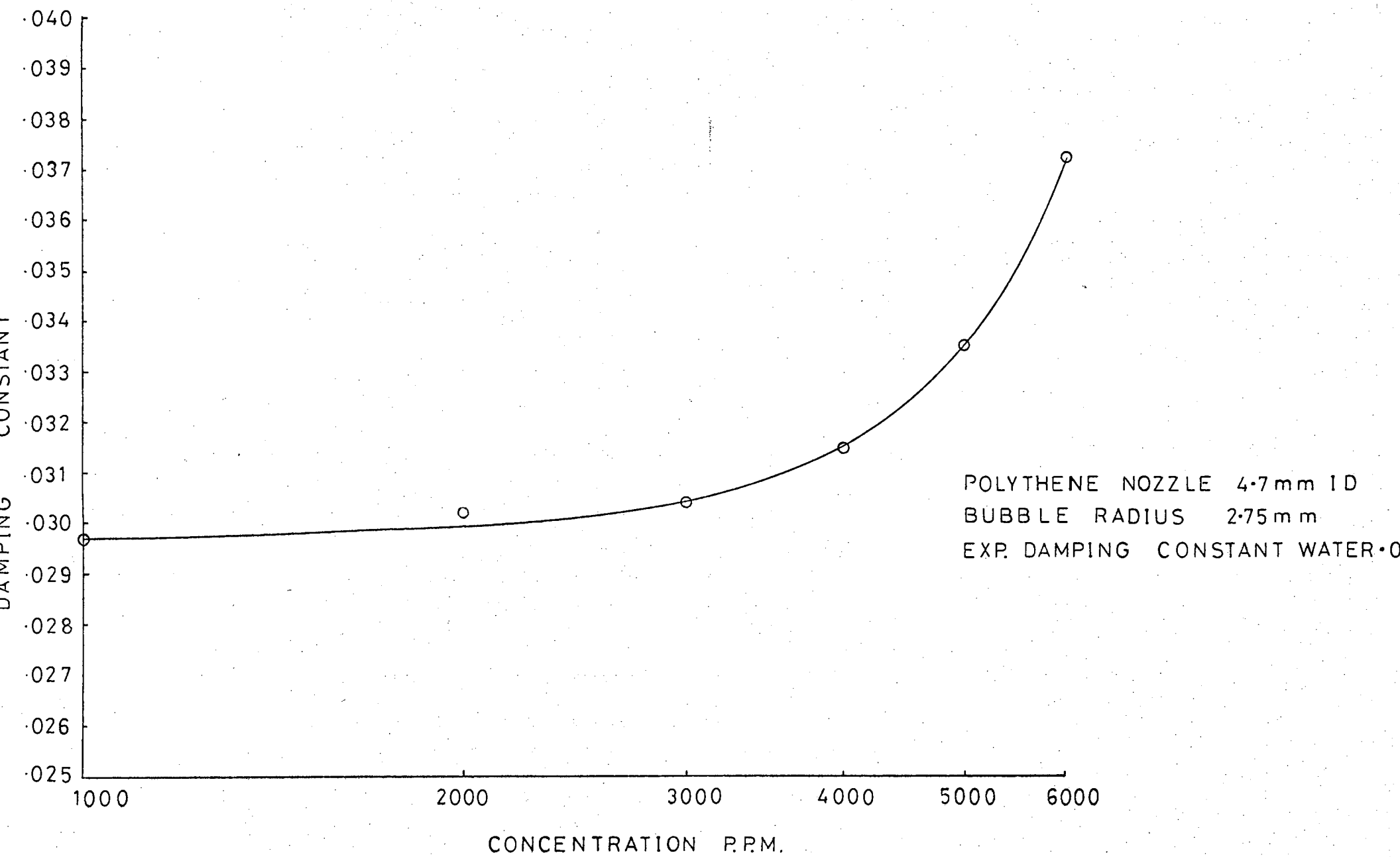


FIG. 8.8 EFFECT OF POLYOX ADDITIVES ON THE DAMPING OF BUBBLE PULSATION.

FIG. 8.9 EFFECT OF POLYOX ADDITIVES ON THE DAMPING OF BUBBLE PULSATION.



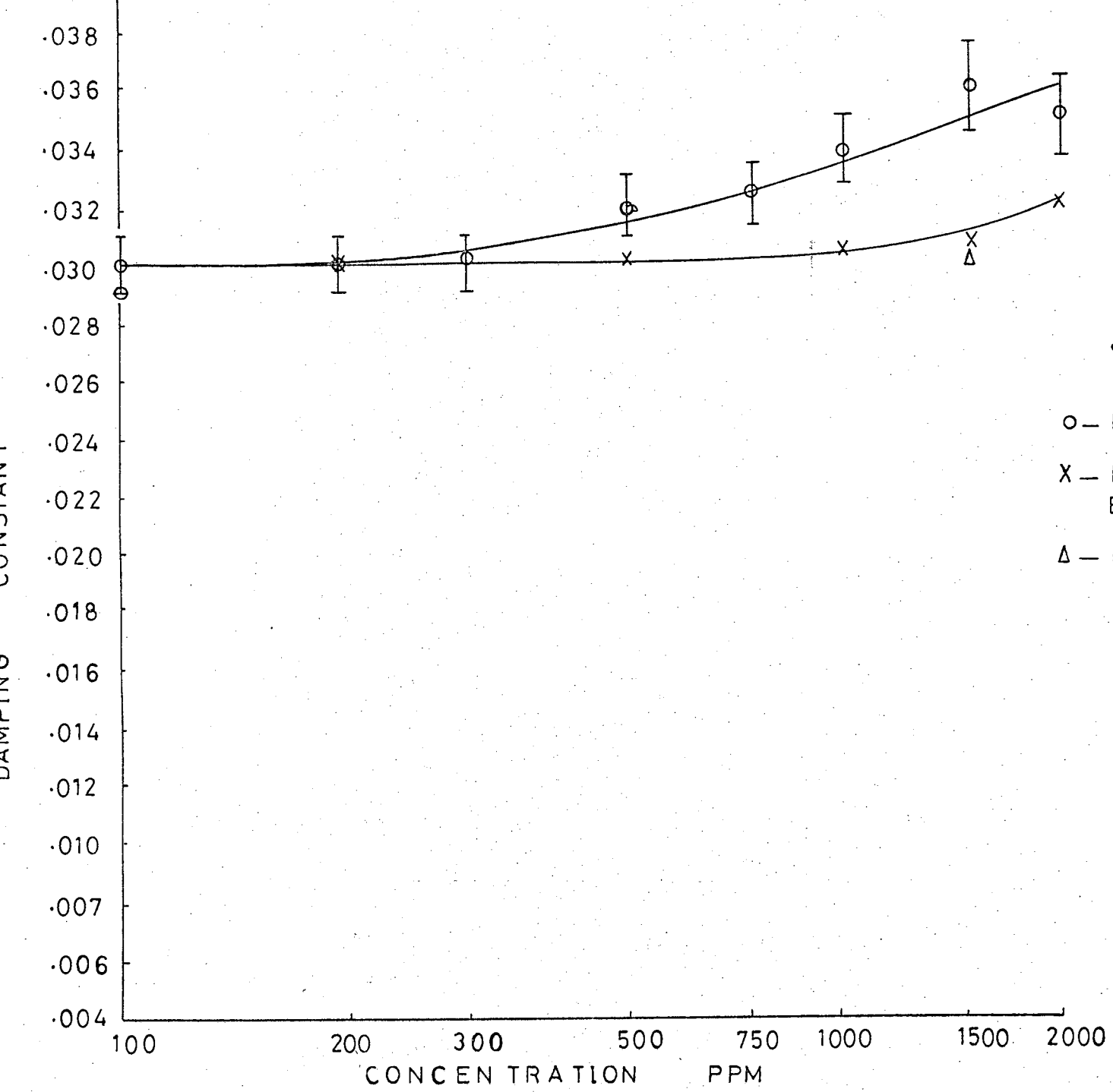


FIG. 8.10 EFFECT OF POLYMER CONCENTRATION ON THE DAMPING CONSTANT OF AIR BUBBLE FREQUENCY.

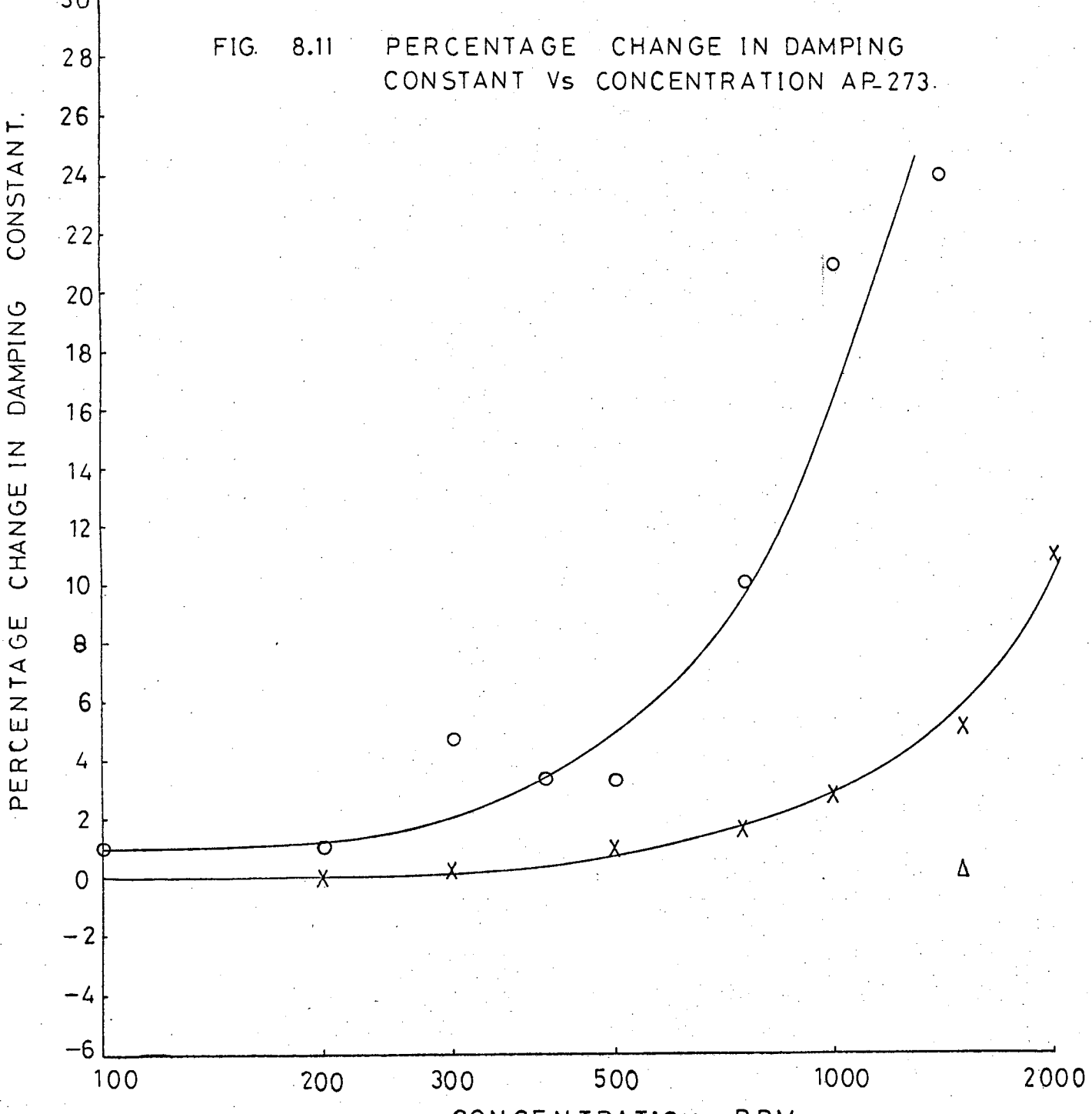
BUBBLE FREQUENCY - 1200 HZ
& EXP. WATER - 0294

o — DAMPING CONSTANT IN AP. 273

X — D.C. IN NEWTONIAN SOLUTION OF EQUIVALENT VISCOSITY

Δ — D.C. IN 0·5 MOLAR Na Cl 1500 AP 27

FIG. 8.11 PERCENTAGE CHANGE IN DAMPING CONSTANT Vs CONCENTRATION AP-273.



X BUBBLE IN NEWTONIAN LIQU
OF EQUIVALENT VISCOSITY.

o AP-273 SOLUTION

Δ 0·5 MOLAR NaCl SOLUTION

POLYTHENE NOZZLE 4·7mm I.D

BUBBLE RADIUS 2·75mm I.D.
(WATER)